Es gibt keine schwarzen Löcher

Von Einstein zu Zweistein

Walter Greiner

FIAS Frankfurt Institute for Advanced Studies

Frankfurt Institute for Advanced Studies

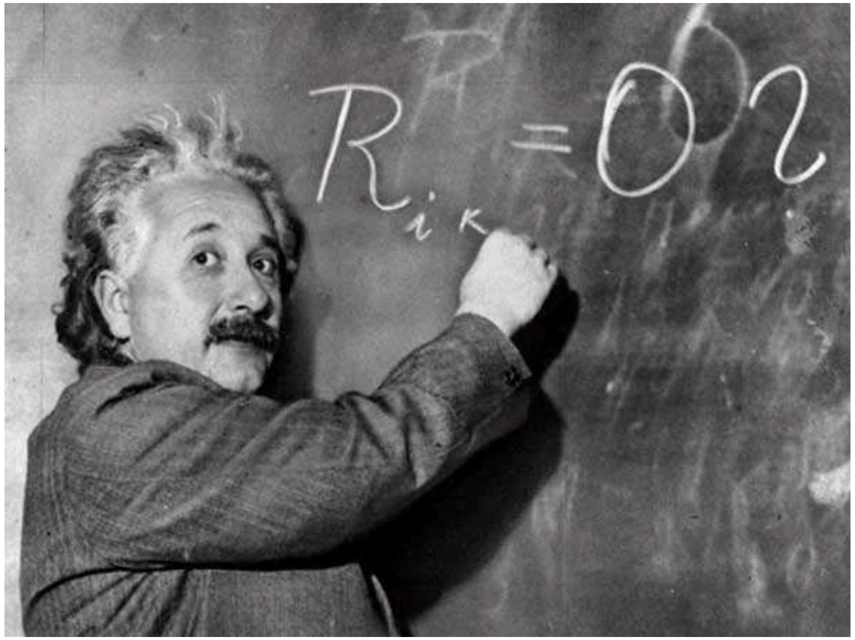
Prof. Senator E. h. Carlo Giersch

Senatorin E.h. Karin Giersch

Special Message to Nicola up in Heaven:

There are no Black Holes!

Albert Einstein



Karl Schwarzschild

Metric:

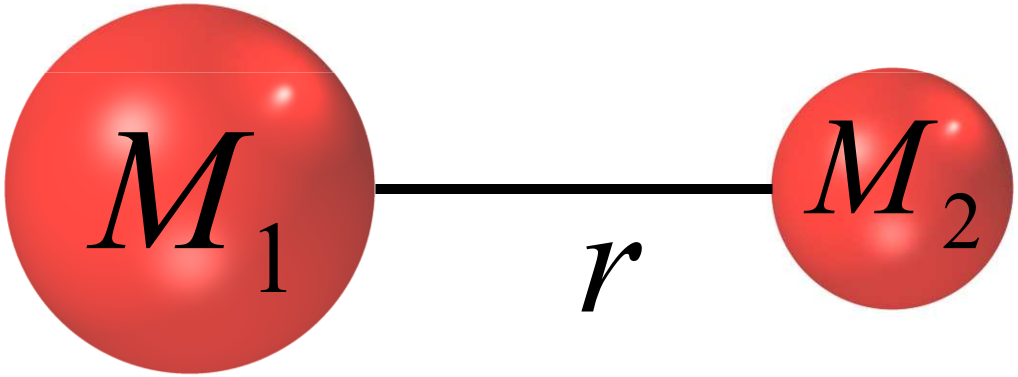
$$d\omega^2 \approx \left(1-\frac{2m}{r}\right)dt^2 - \frac{dr^2}{1-\frac{2m}{r}} - r^2\left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

<u>Singularity:</u> light cannot escape from within the Schwarzschild radius

Astronaut

A fatal fall into the gravitational center (tidal forces)

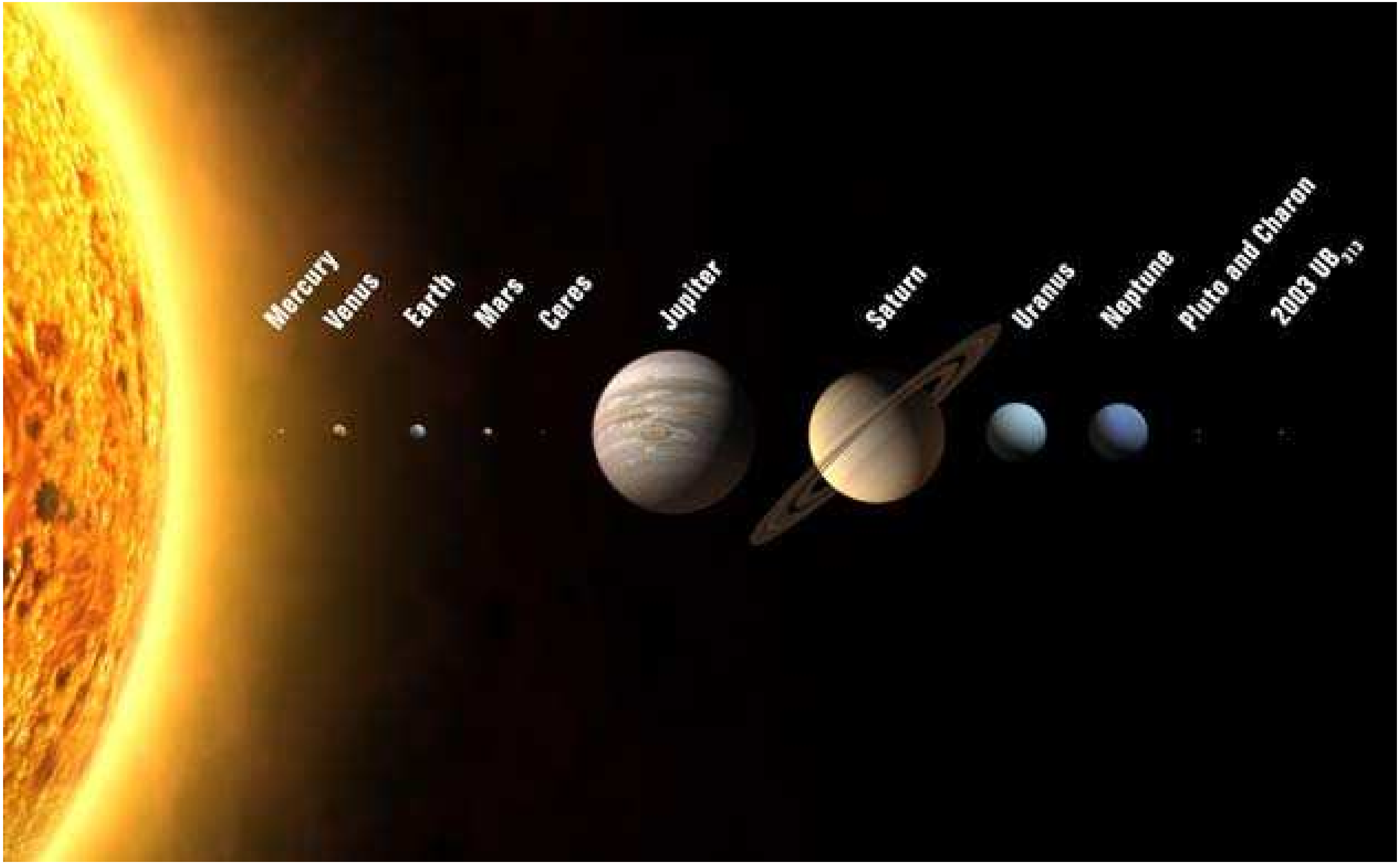
Gravitationskraft



$$K = \gamma \frac{M_1 M_2}{r^2}$$

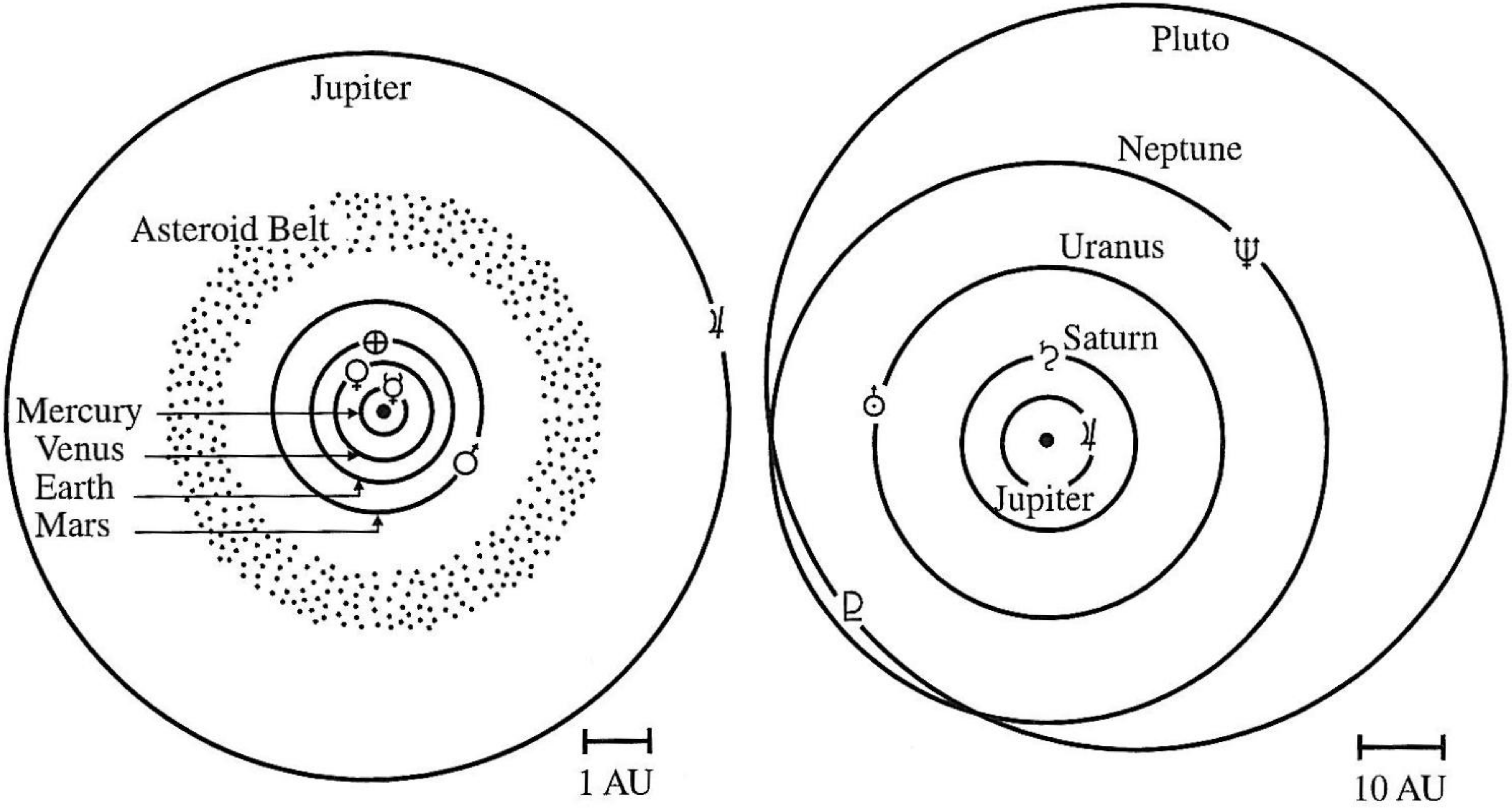
Isaac Newton
*** 04.01.1643-**
† 31.03.1727

Relative Größen von Sonne und Ihren Planeten

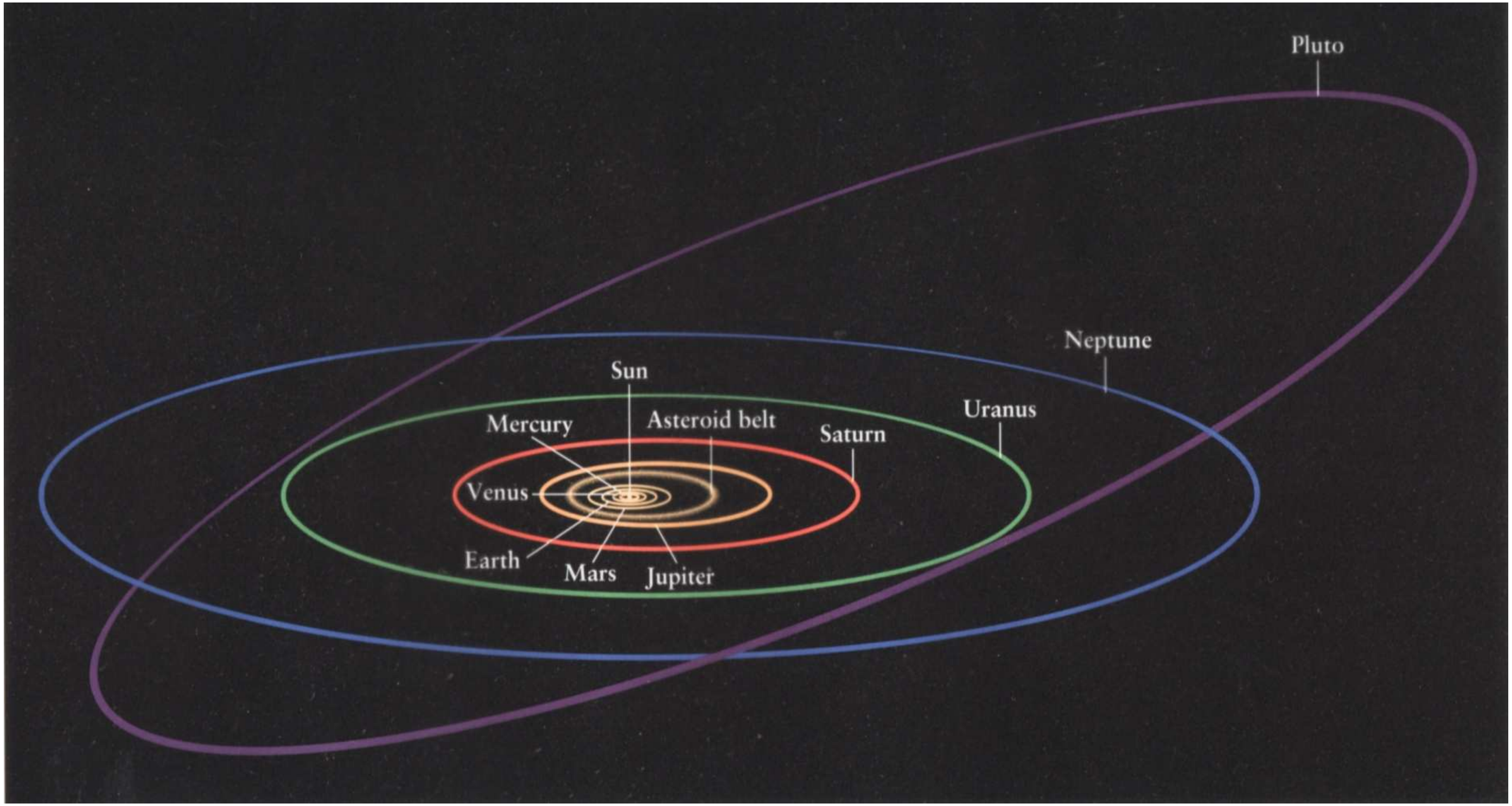


Sonnenradius ≈ 695000 km **Erdradius ≈ 6400 km**

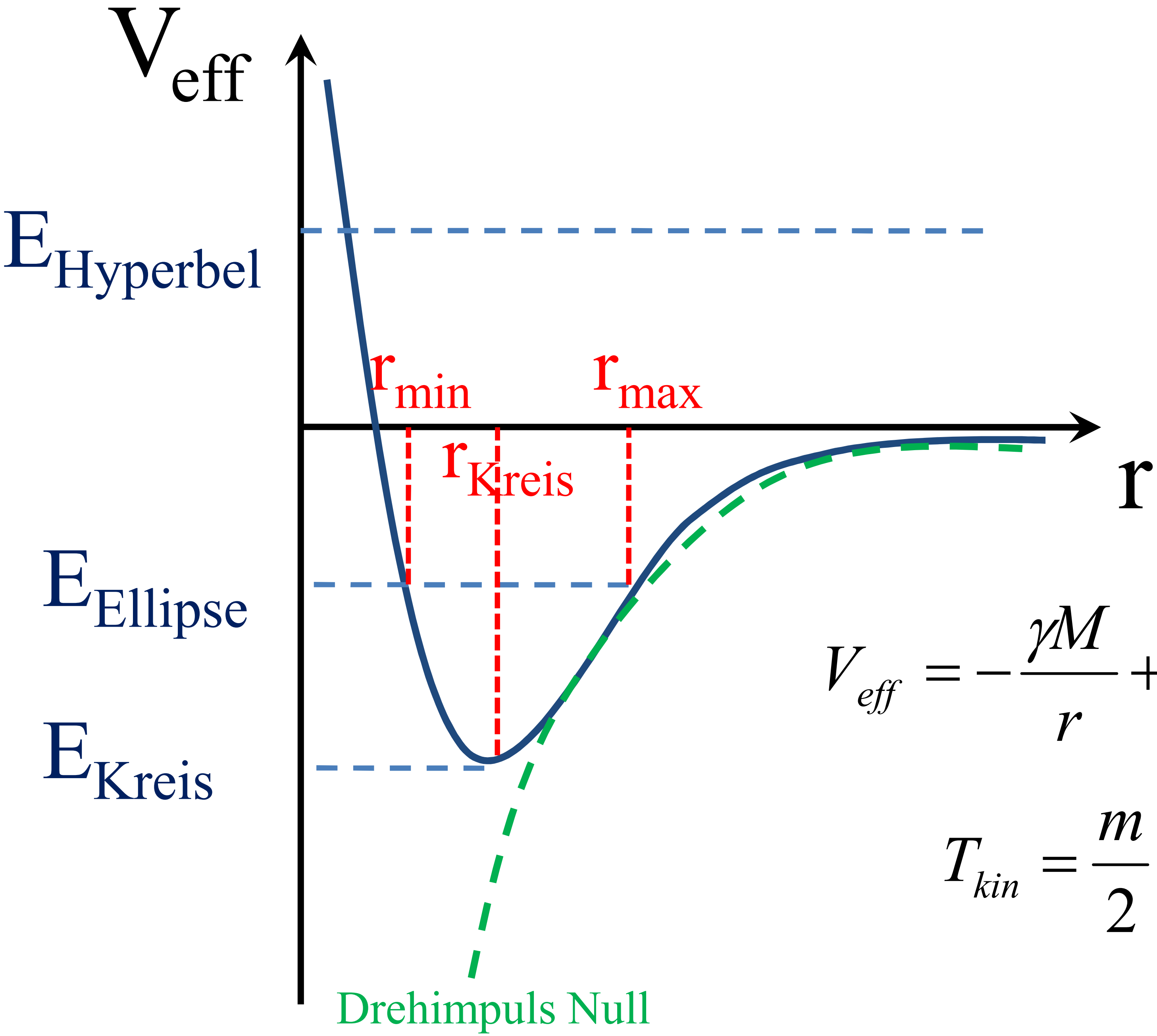
Planeten und deren Umlaufbahnen



Solar System



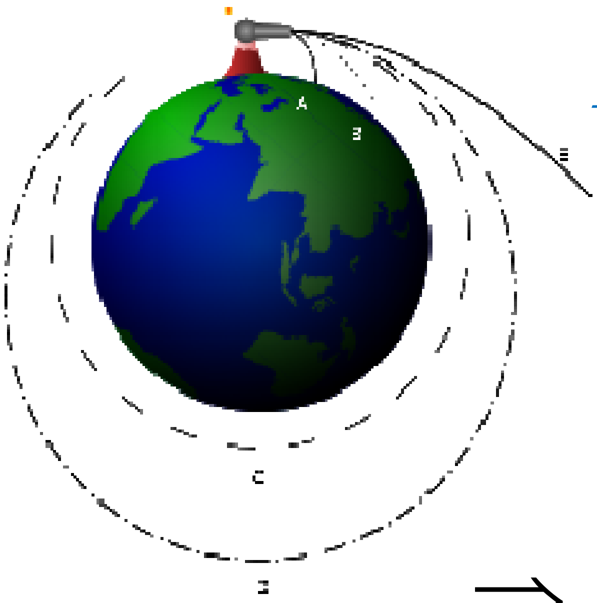
Bahntypen im effektiven Potential



$$V_{eff} = -\frac{\gamma M}{r} + \frac{L^2}{2mr^2}$$

$$T_{kin} = \frac{m}{2}\dot{r}^2$$

Fluchtgeschwindigkeit



Die Gesamtenergie ist gleich auf der Erdoberfläche und weit weg von der Erde

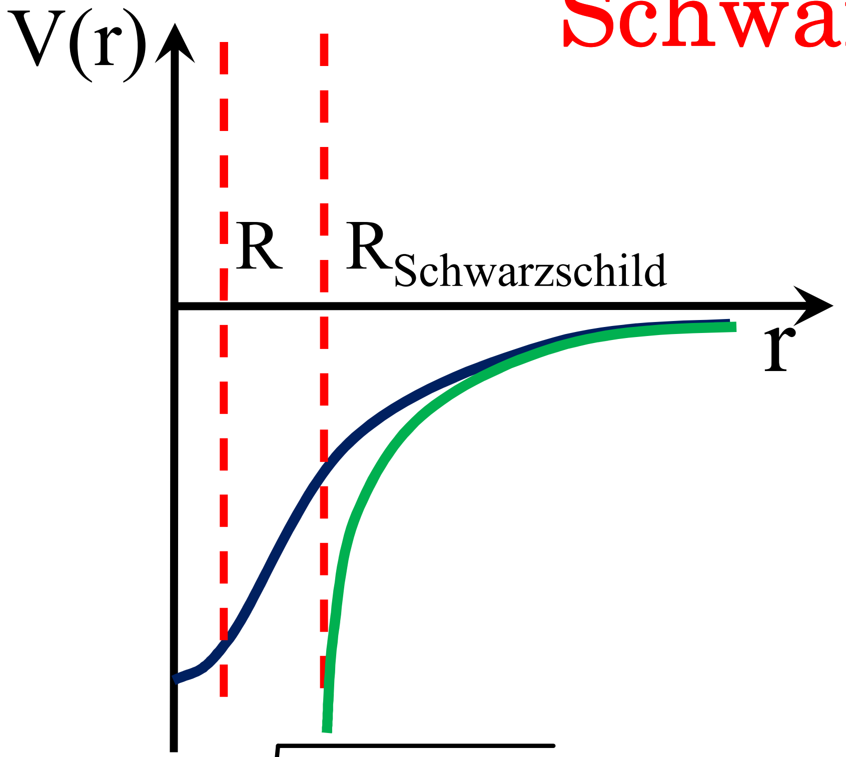
$$E = \frac{1}{2} m V_F^2 - \frac{\gamma M_\oplus m}{R_\oplus} = E(r \to \infty) \cong 0$$

$$\Rightarrow V_F = \sqrt{\frac{2\gamma M_\oplus}{R_\oplus}}$$

ist die kleinste nötige Geschwindigkeit um Gravitationsfeld verlassen zu können.

Himmelskörper	Fluchtgeschwindigkeit km/s
Erde	11.2
Venus	10.2
Jupiter	59.6
Sonne	617.3

Schwarzschild Radius

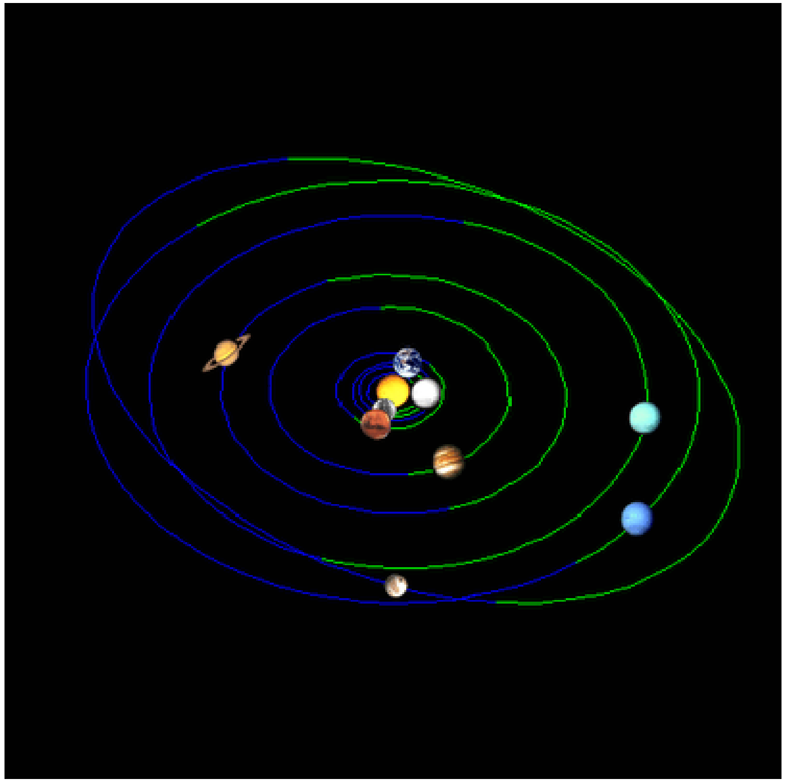


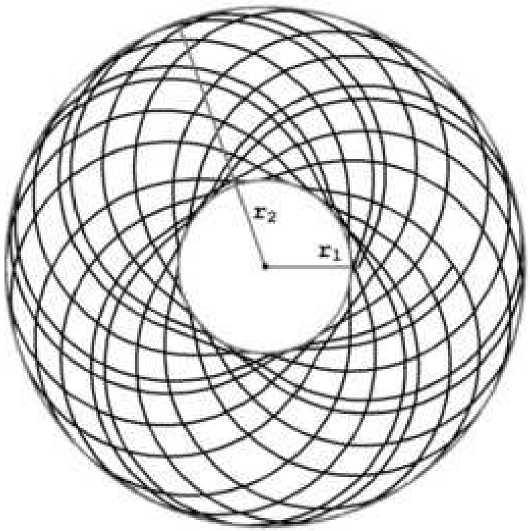
Naive Rechnung:
die Fluchtgeschwindigkeit darf die Lichtgeschwindigkeit nicht überschreiten

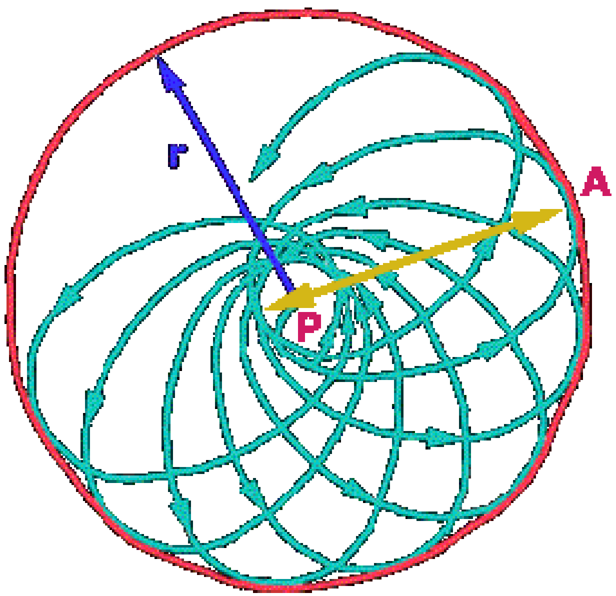
$$V_F = \sqrt{\frac{2\gamma M}{R}} = c \quad \Rightarrow \quad R_{Schwarzschild} = \frac{2\gamma M}{c^2}$$

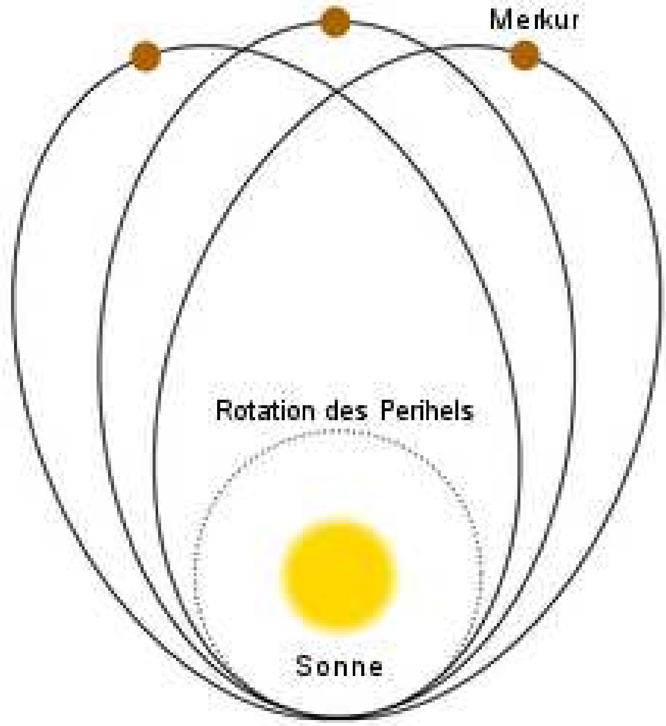
Himmelskörper-Masse	Schwarzschild Radius
Erde	1 cm
Jupiter	3.2 m
Sonne	3 km
Syrius A	6.3 km

Geschlossene Umlaufbahnen und Rosettenlaufbahn

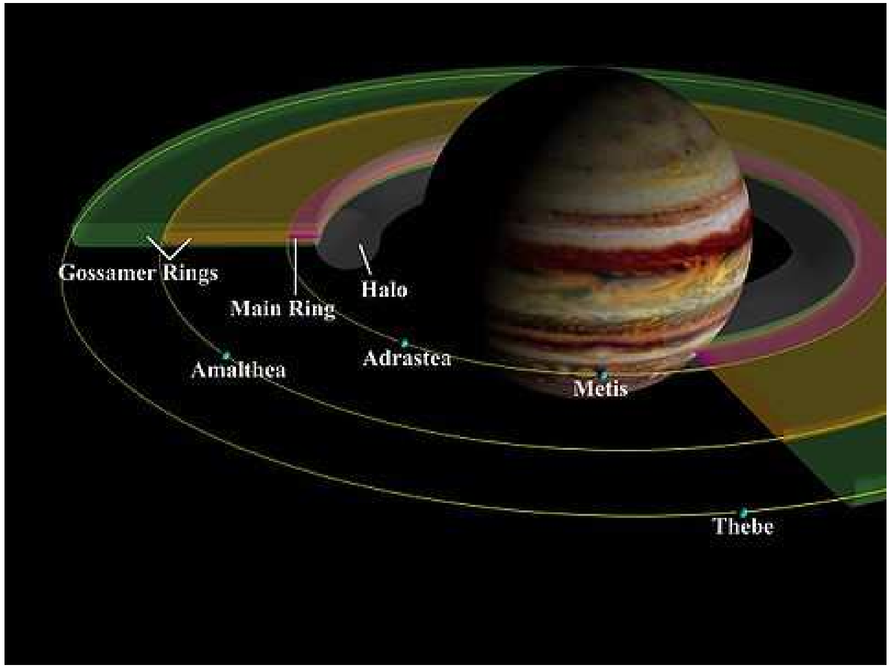


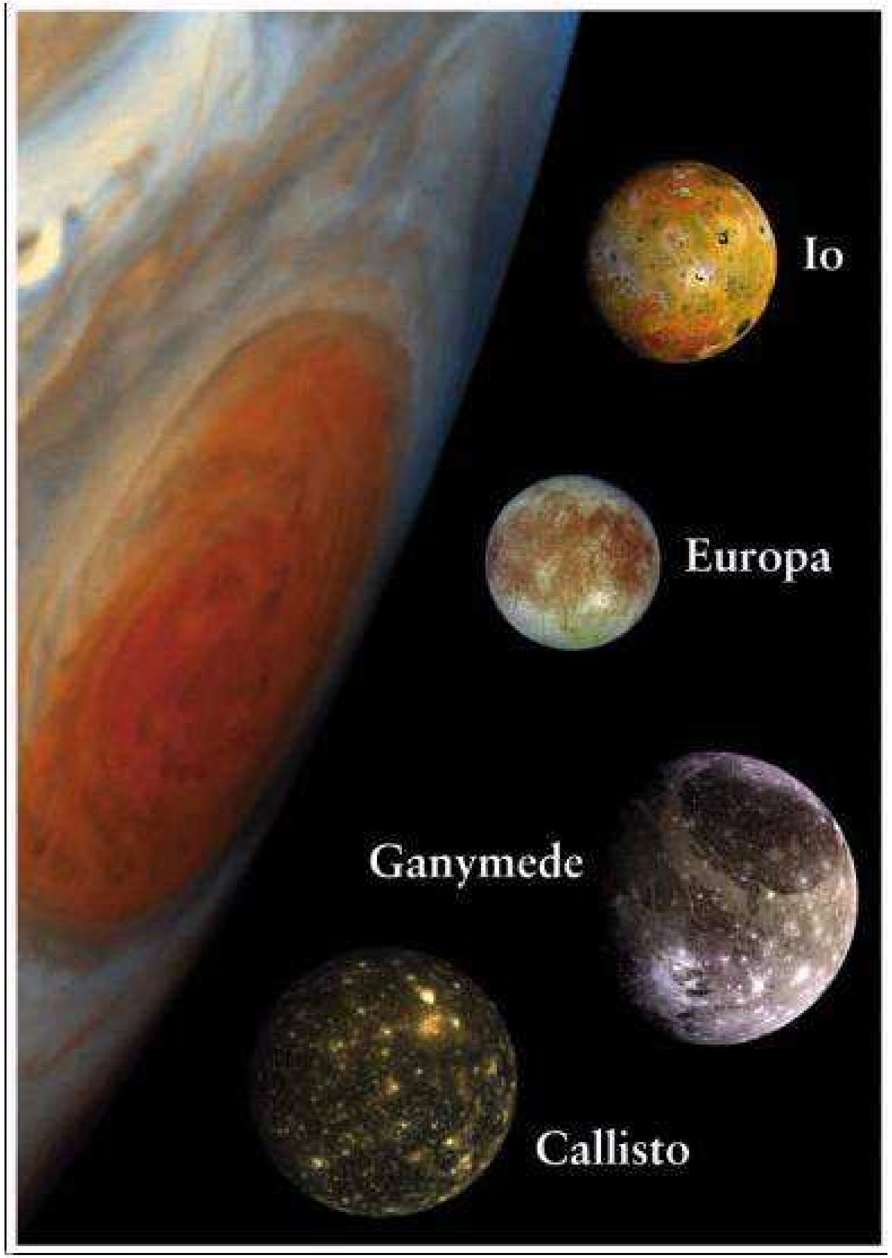


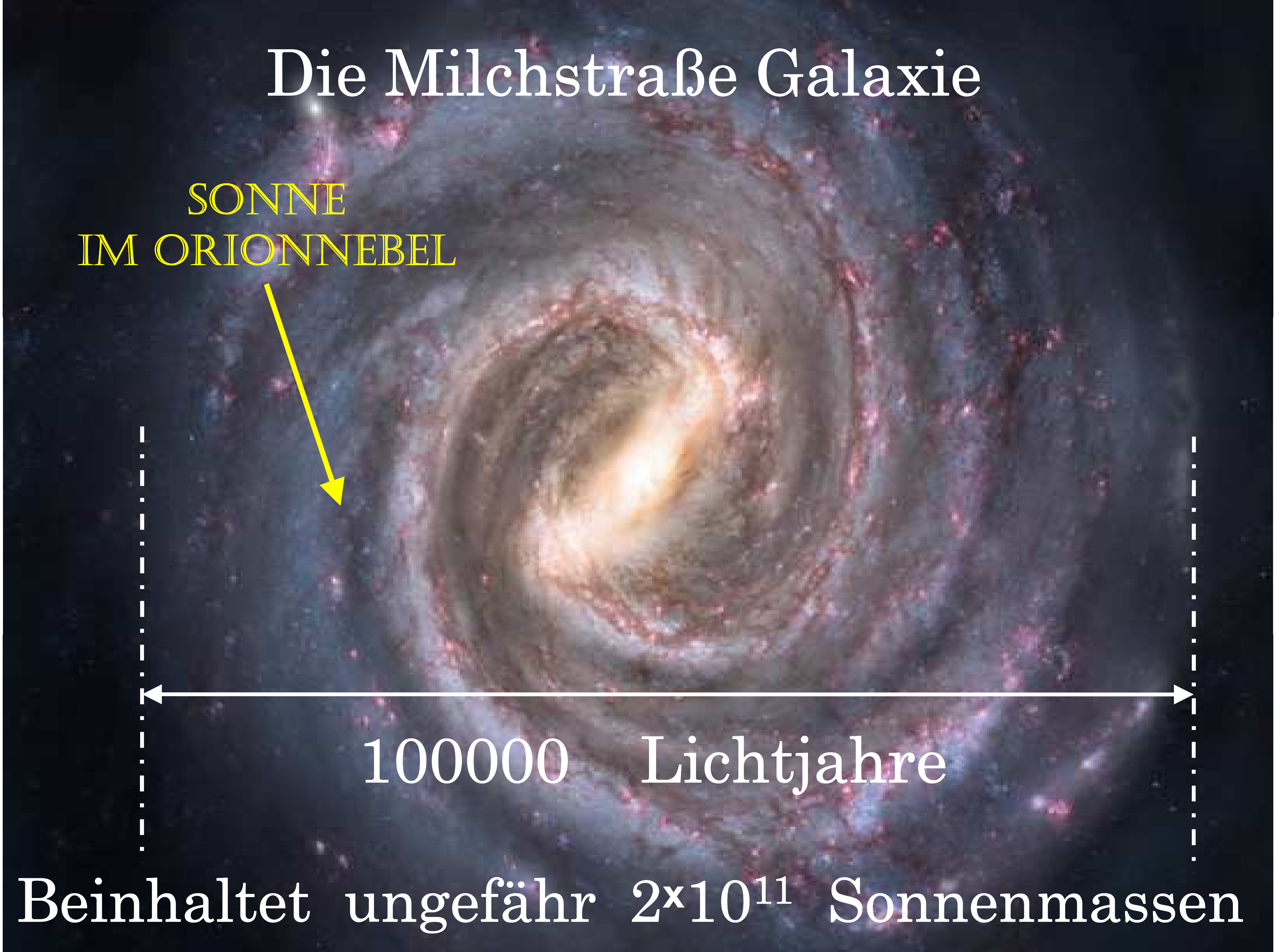




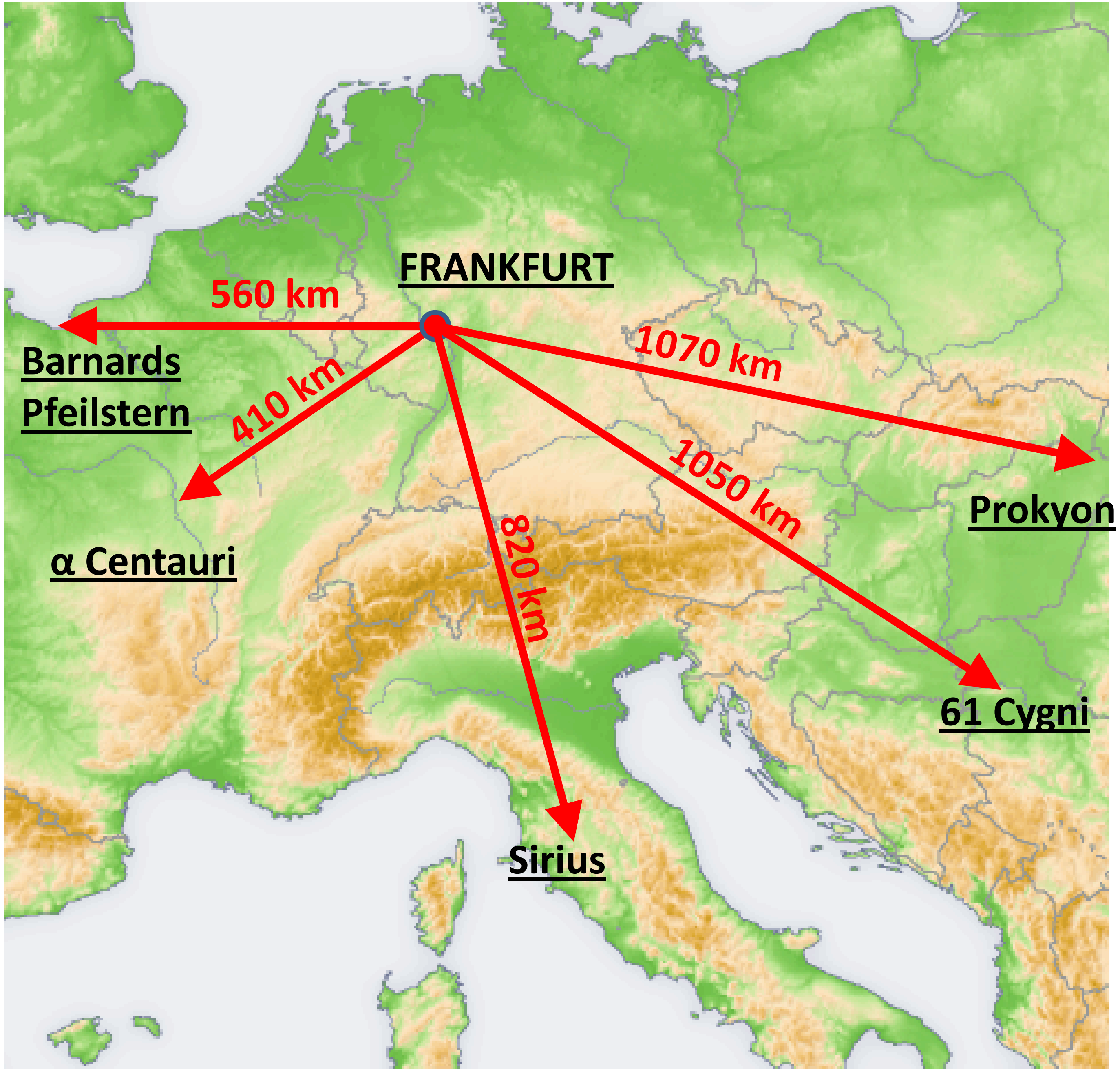
Jupiter Moons



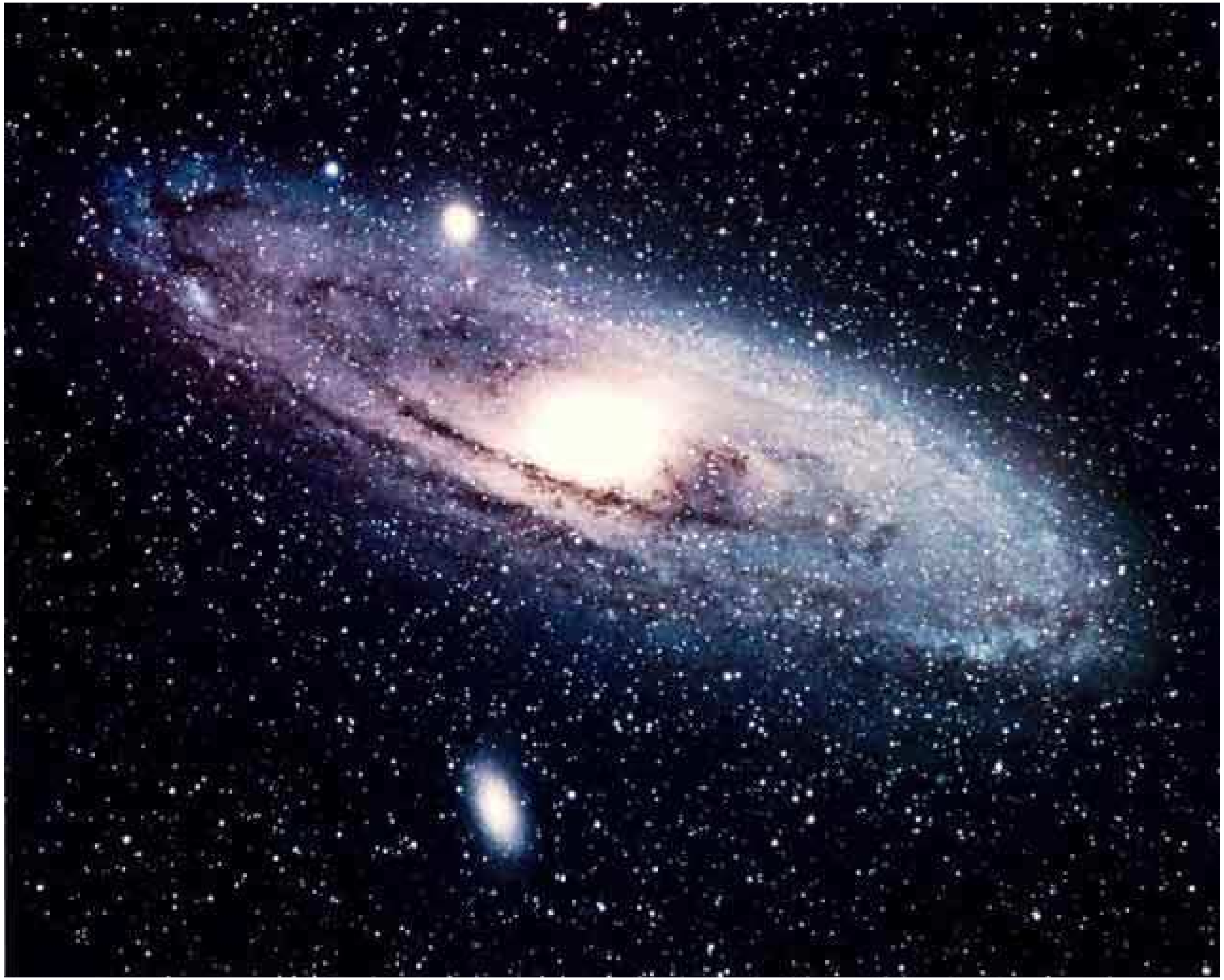



Die Milchstraße Galaxie
SONNE
IM ORIONNEBEL
100000 Lichtjahre
Beinhaltet ungefähr 2×10^11 Sonnenmassen

Die nächste Umgebung der Sonne



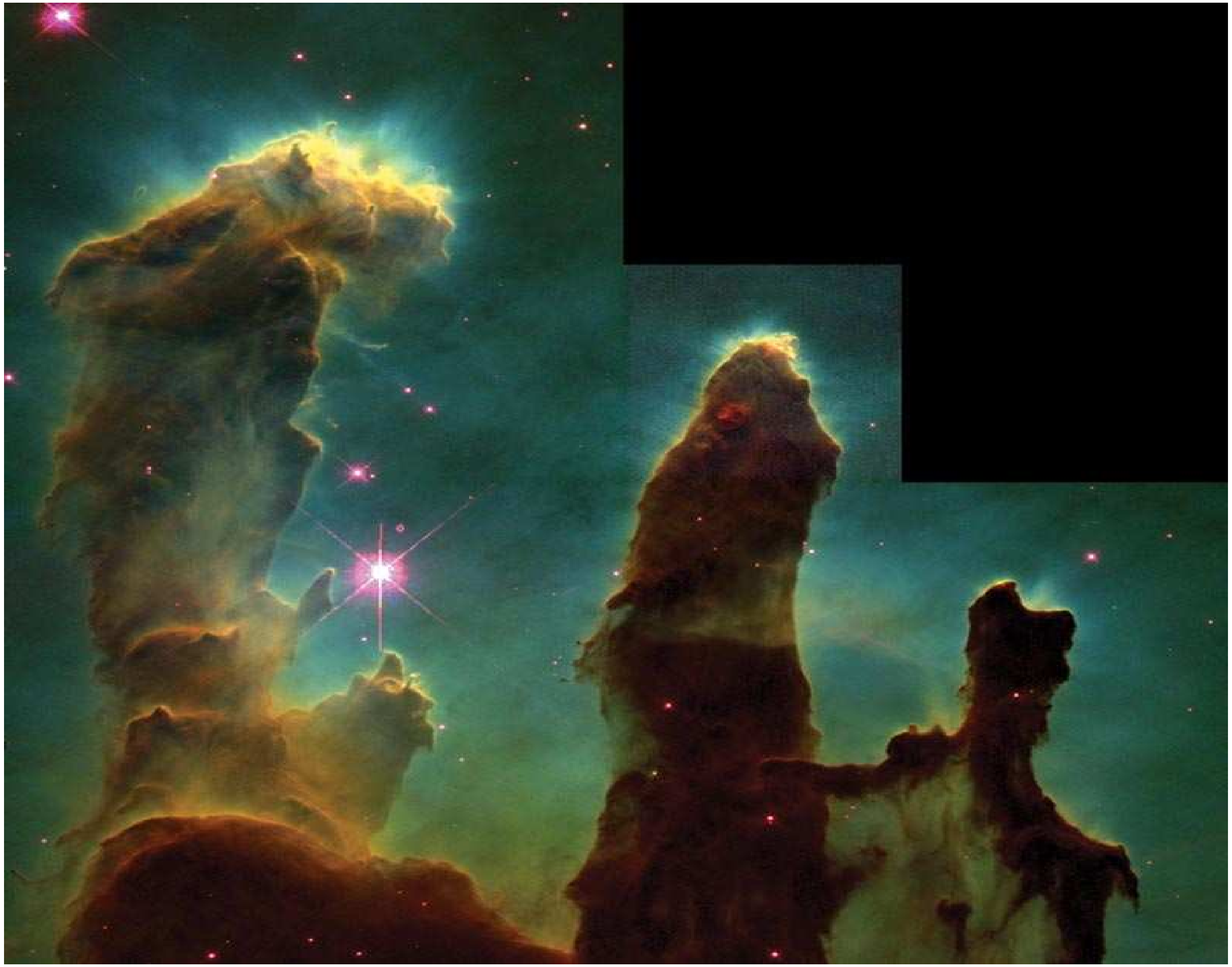
Die Milchstraße



Magellansche Wolken kreisen um Milchstraße

This galaxy known as Messier101 (Feuerrad-Galaxie)
helped measuring the expansion rate
of the universe within the Hubble project

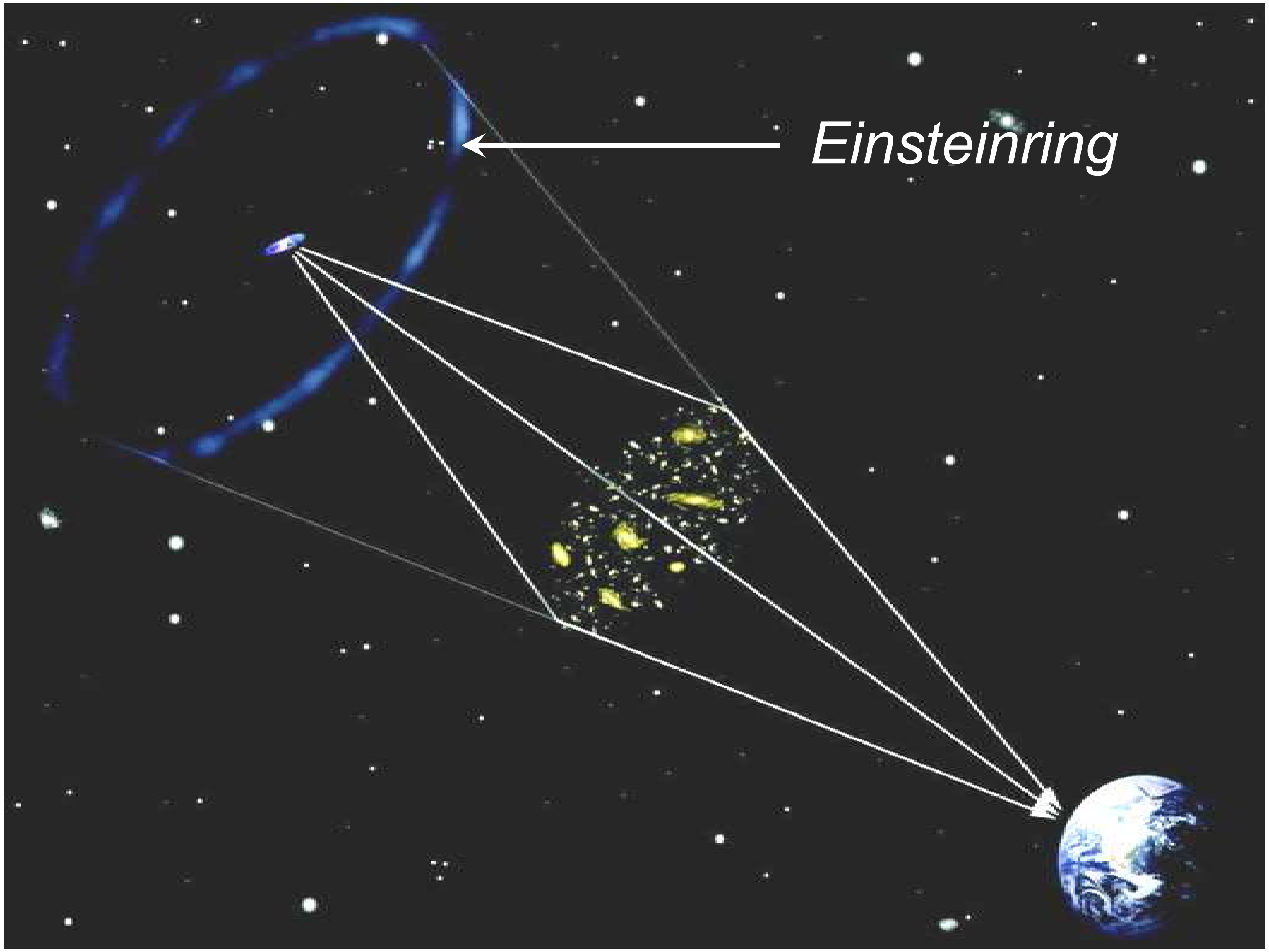
Interstellar gases are collecting into newly forming stars



Sometimes they reach 100 LY in diameter and have the mass of 6×10^6 solar masses (T=10 K).

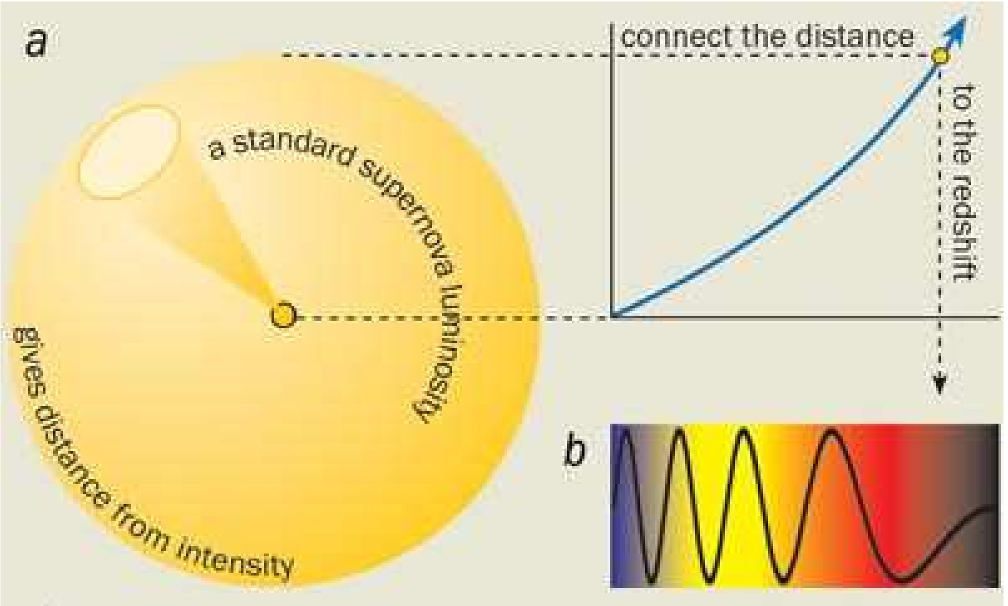
"Eye of God“ - the Helix Nebula is actually the glowing gas remnants of a dying star

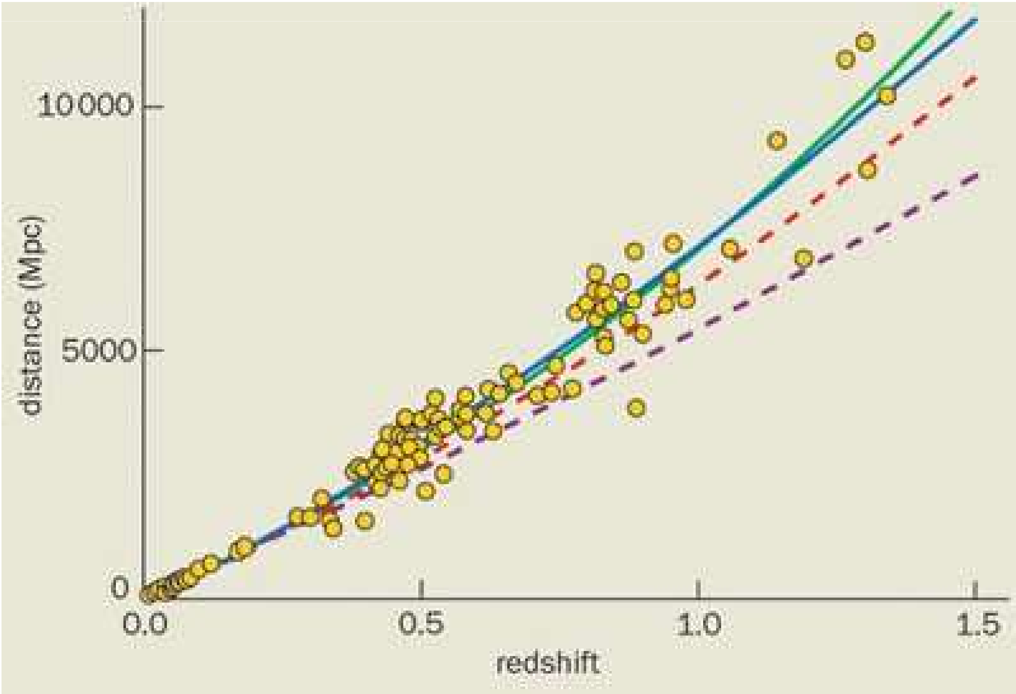
Gravitationslinse



Beschleunigte Expansion des Universums in Supernovae Entfernungsmessungen.

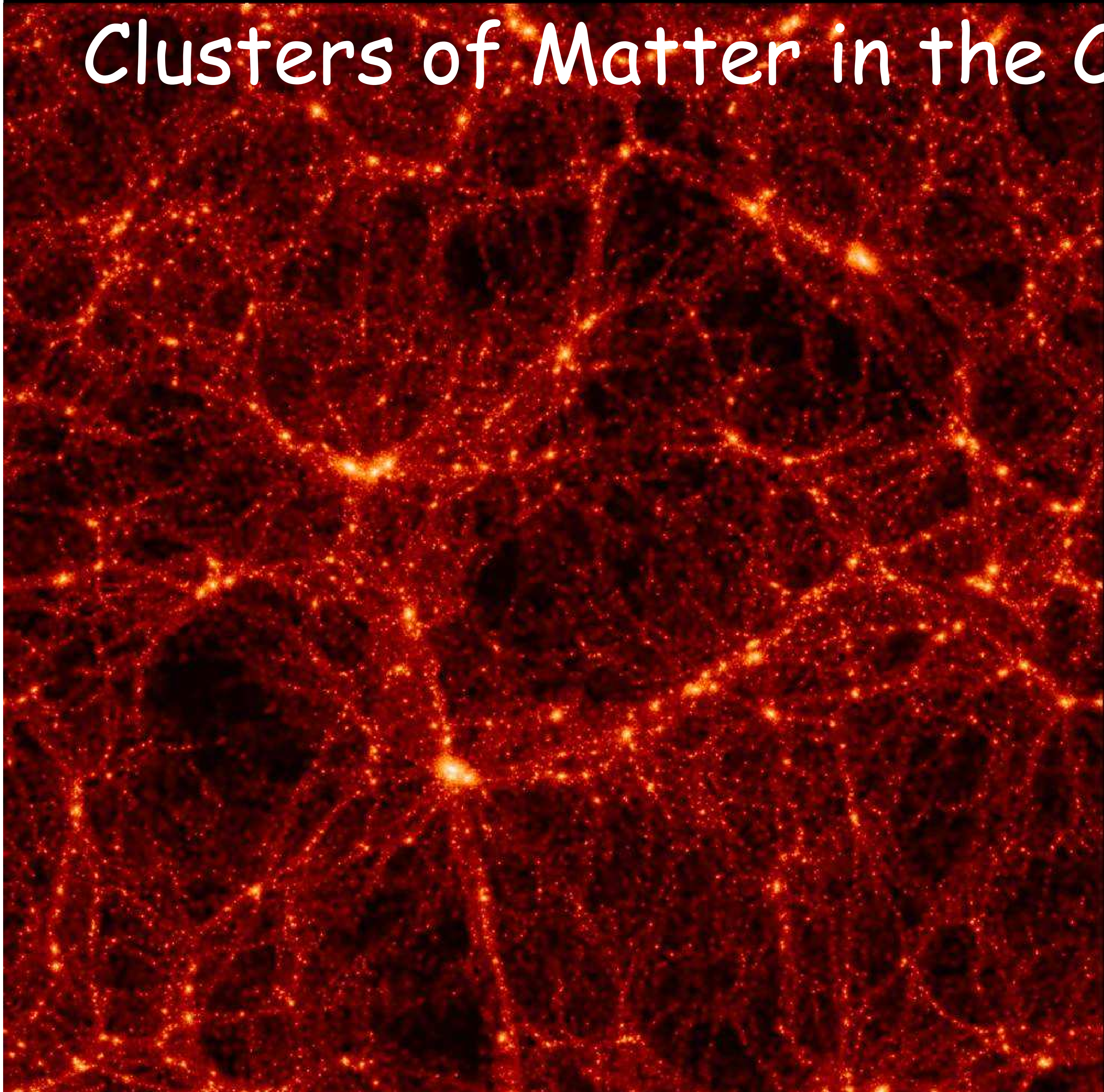
Dunkle Materie?





Blaue Linie zeigt die Messungen. Grüne Linie entspricht dem Weltraum, der sich weder beschleunigt noch verzögert. Andere Linien zeigen aktuelle theoretische Modelle.

Clusters of Matter in the Cosmos



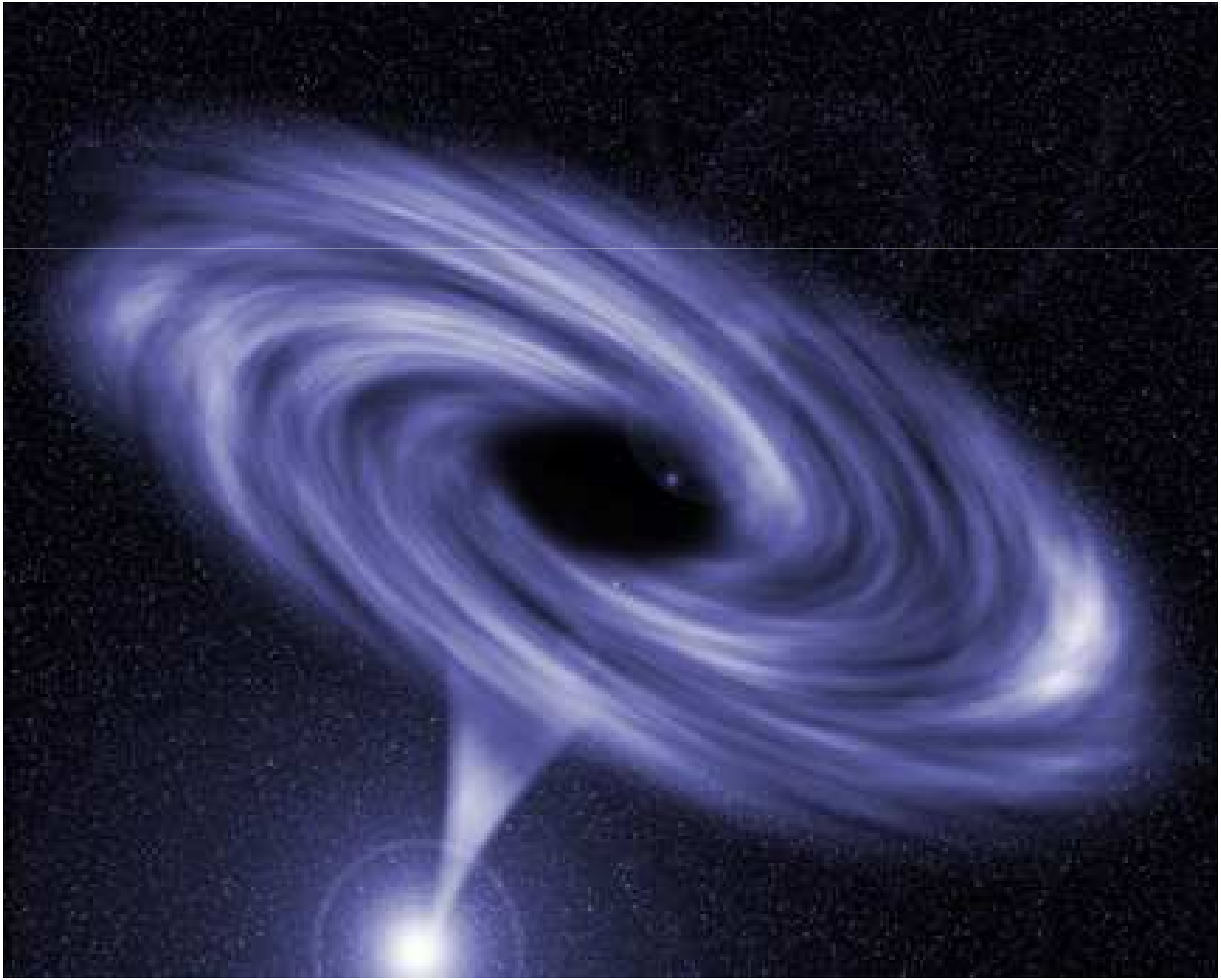
DARK ENERGY:

ANTI-Gravitation!
-Drives the Universe apart:

Expansion

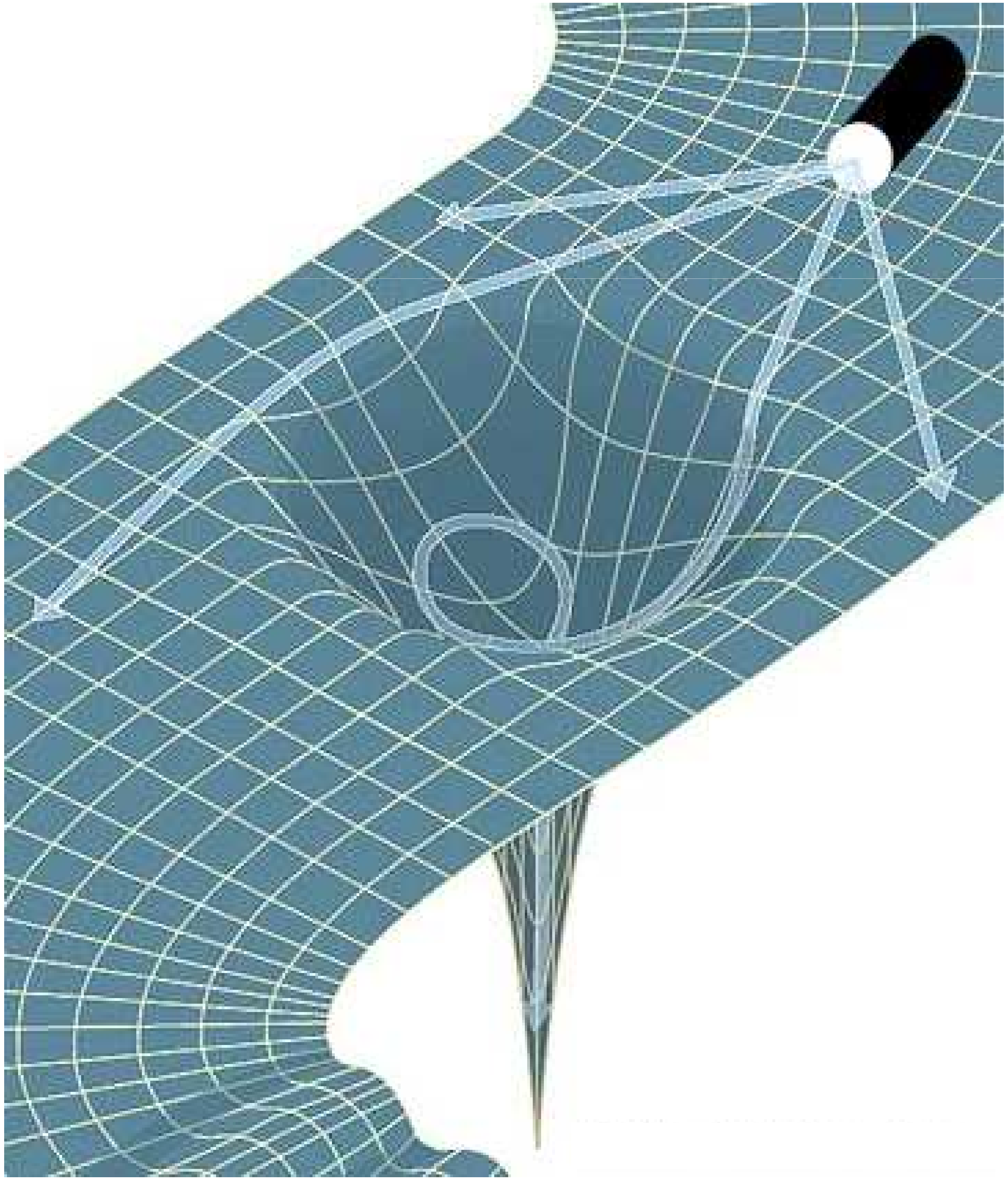
Large - scale cosmic matter distribution (simulation)

“Schwarzes Loch”



“Schwarzes Loch”

“Schwarzes Loch”



Gibt es Schwarze Löcher?

Erde → Schwarzes Loch? Nein!
Erd-Masse mit Durchmesser kleiner als 2 cm!

Sonne → Schwarzes Loch? Nein!
Sonnen-Masse mit Durchmesser kleiner als 6 km…

Jedoch: Im Zentrum unserer Milchstraße sitzt ein gewaltiges Schwarzes Loch von M = 3,6 Millionen Sonnenmassen !

Ein Monster im Sternbild Schütze:

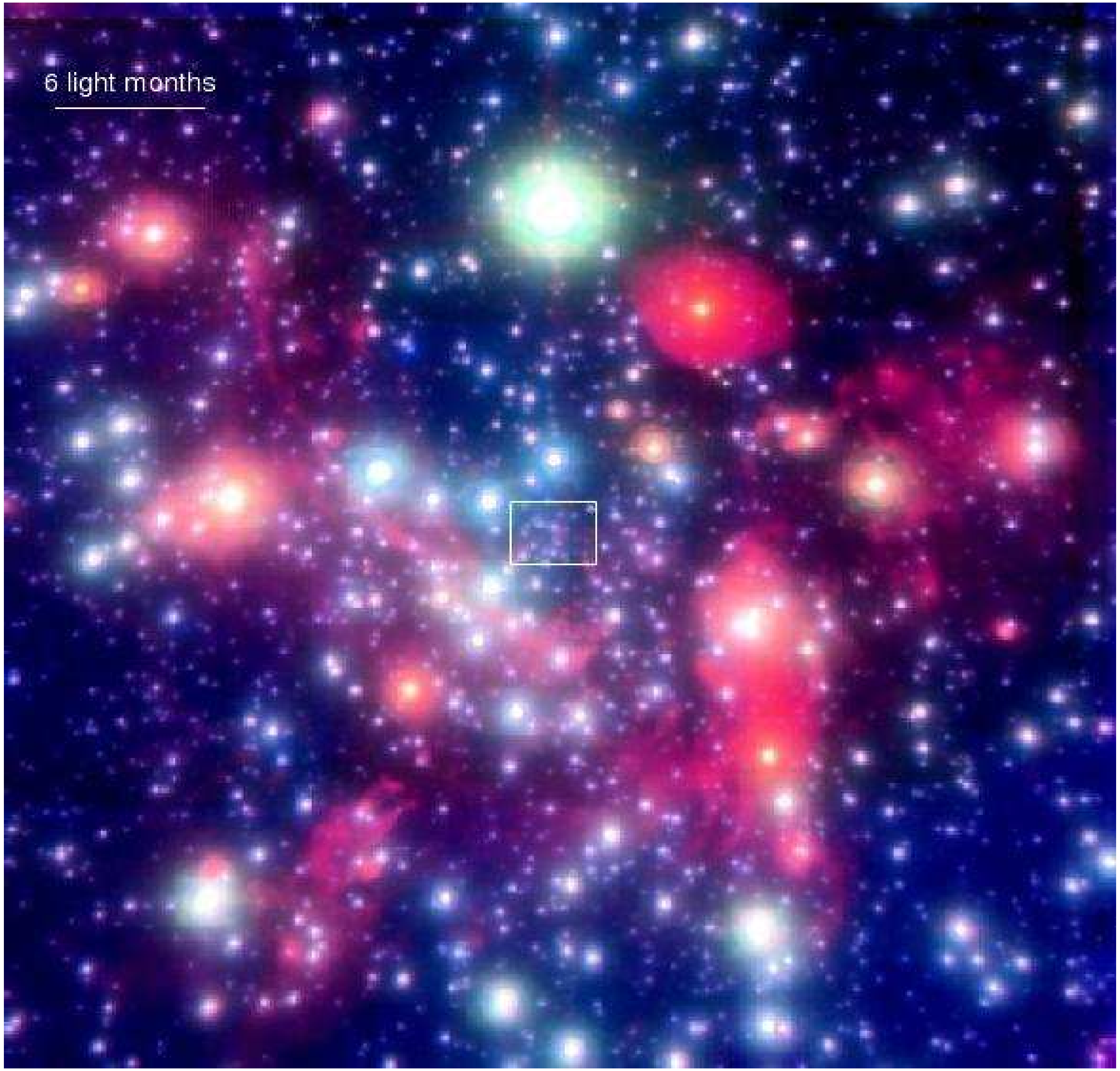
wurde entdeckt von Reinhard Genzel aus München (MPI für Astrophysik)

Ein Schwarzes Loch von

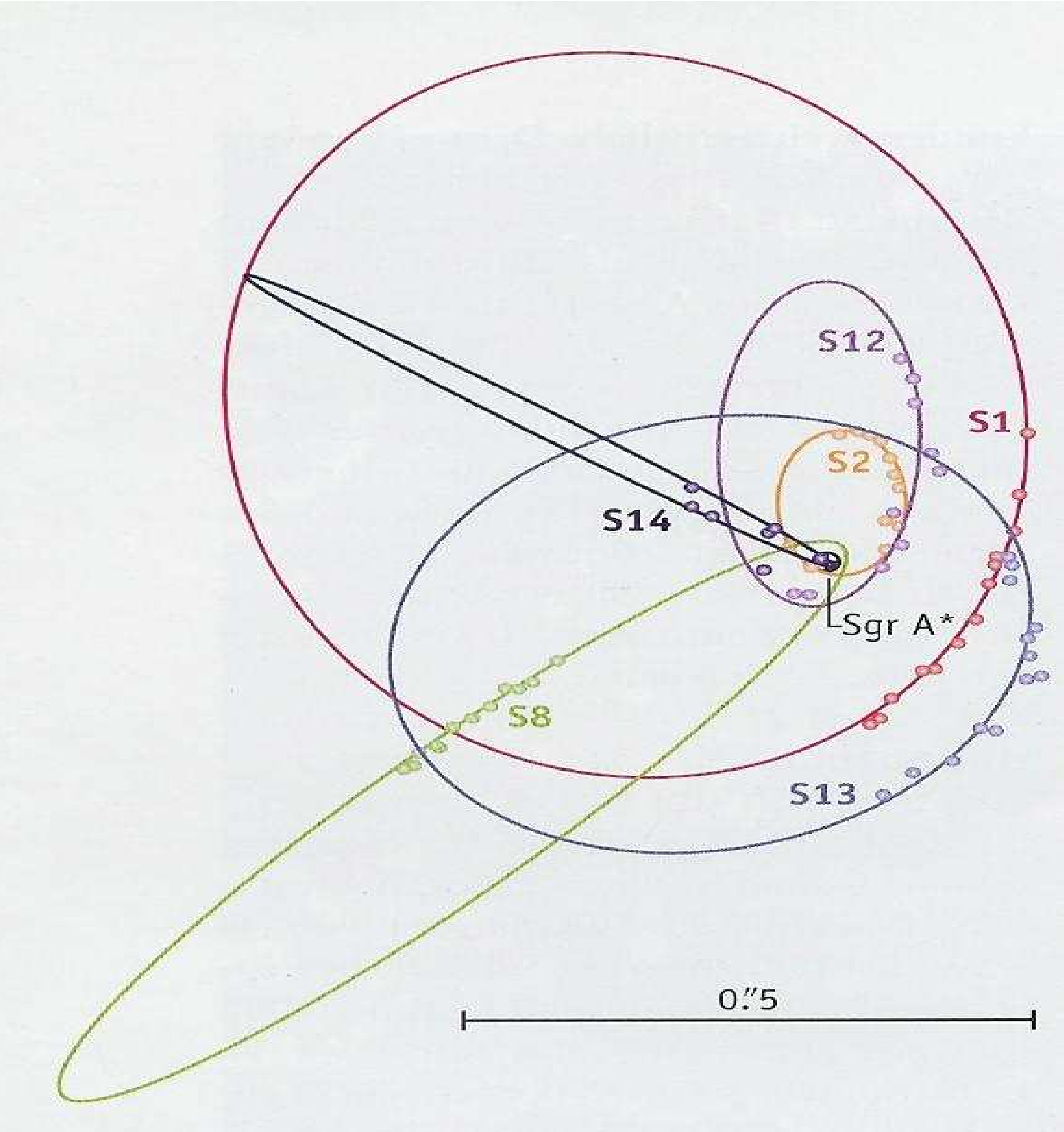
M =3,6 Millionen Sonnenmassen im Zentrum unserer Milchstraße

Bewegung von Sternen um das Schwarze Loch beobachtet !

Schwarzes Loch im Zentrum unserer Milchstraße M=3,6 Millionen Sonnenmassen



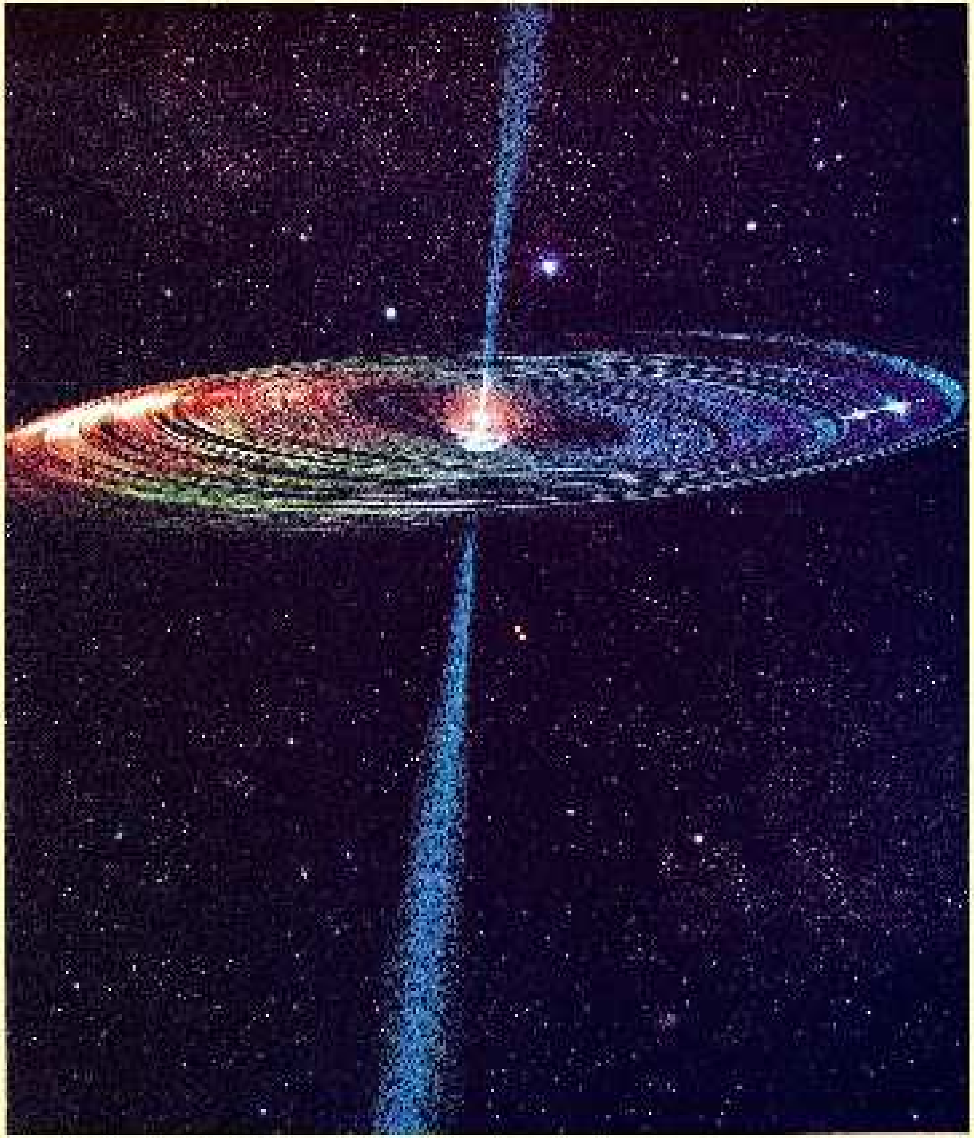
Bahnen von Sternen 1992 – 2006 innerhalb von 10 Lichttagen um das Zentrum der Milchstraße Sagitarius A*, Sternbild Schütze: Genzel et al.



Das Zentrum der Milchstraße ist 25 000 Lichtjahre von der Sonne entfernt.

Sichtbares Licht wird an Staub absorbiert.

Wir können bis zum Zentrum mit infrarotem Licht sehen.



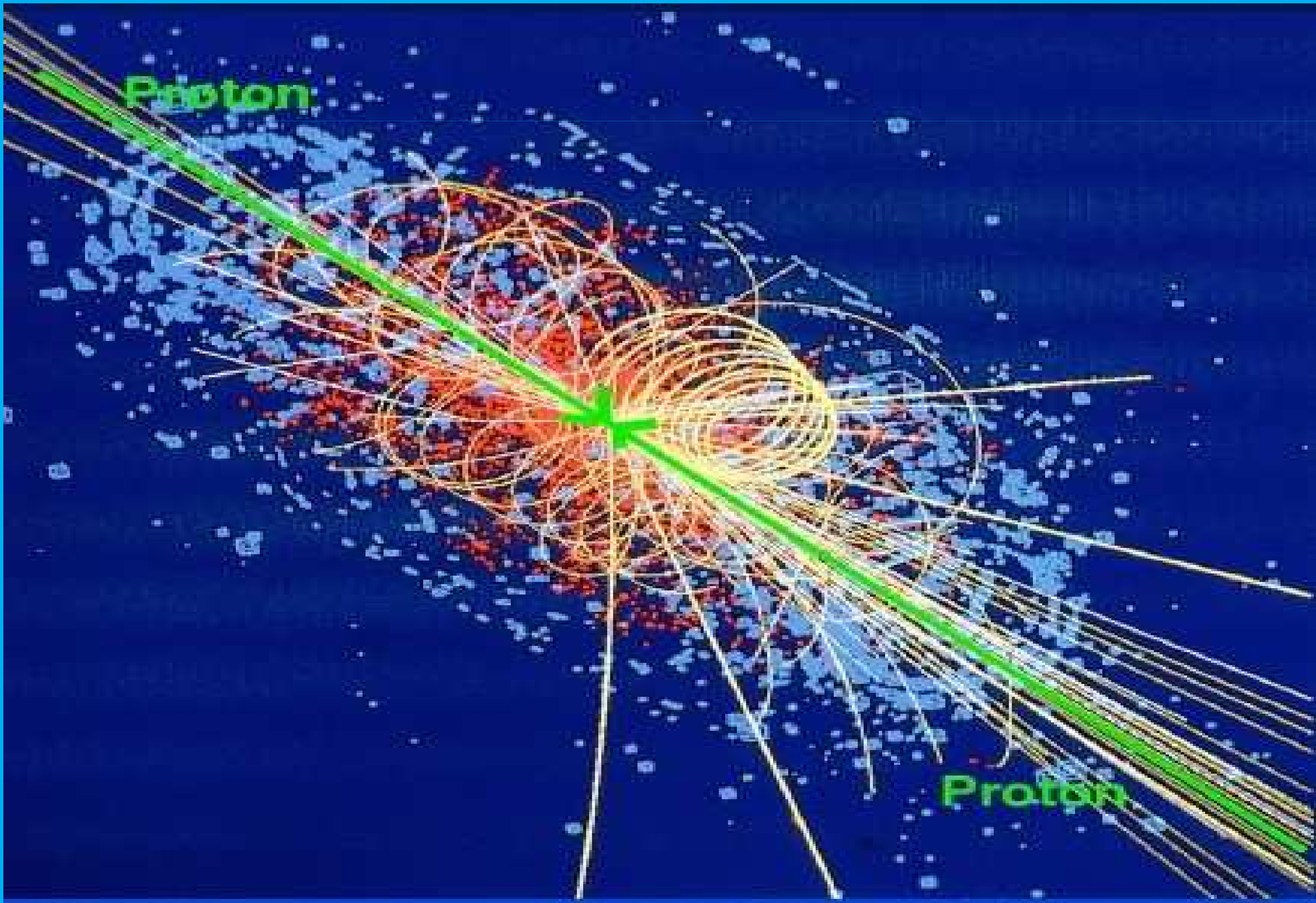
Aufbau der aktiven Galaxiekerne
ähnlich wie galaktische scharze Löcher,
aber insgesamt etwas beeindruckender...

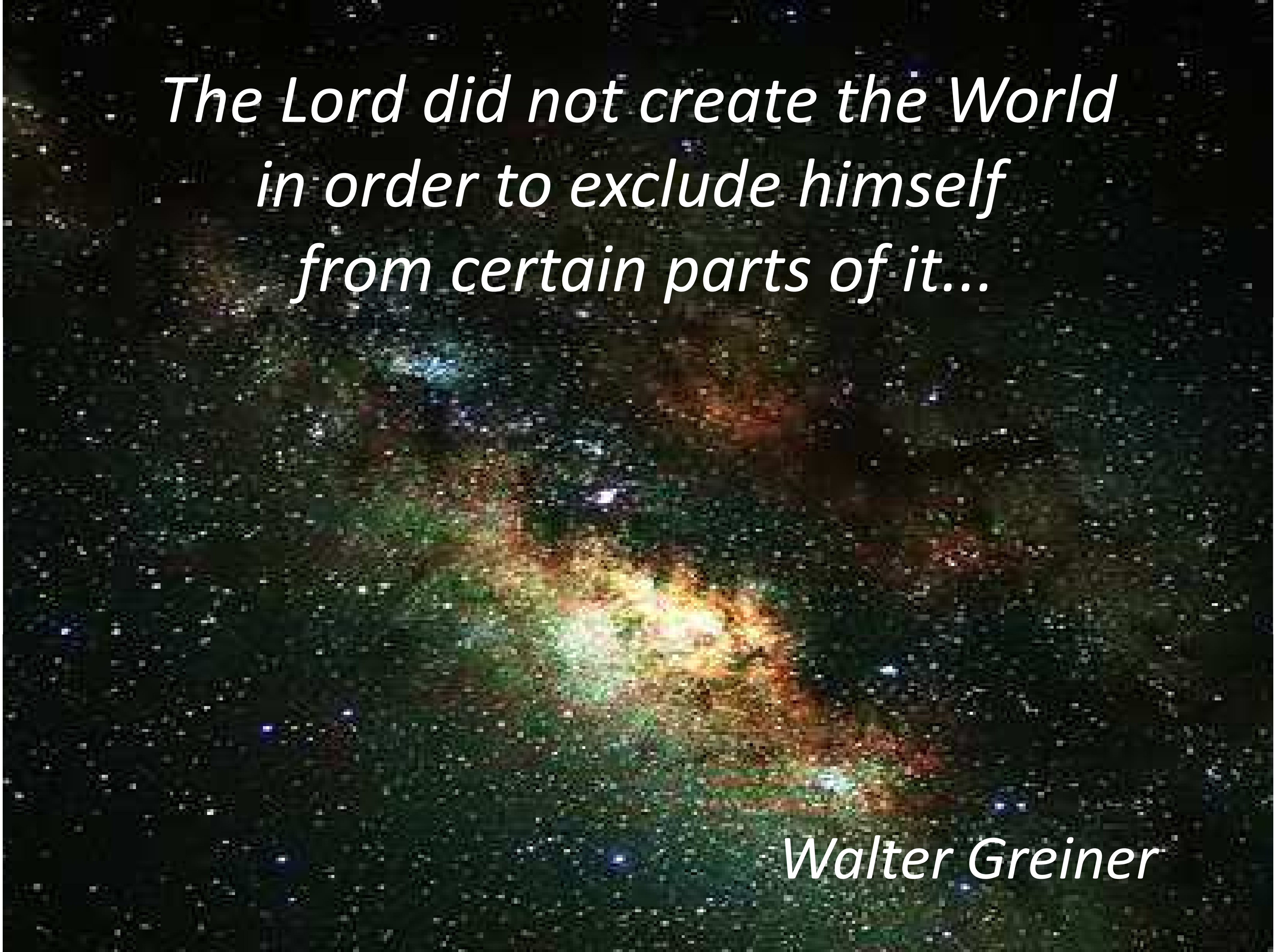
- supermassives schwarzes Loch (10^7 $M_{\odot}$)
- Akkretionsrate der Scheibe (1-2 $M_{\odot}$/Jahr)
- Leuchtkraft hoch ($L \approx 10^{10}$ $L_{\odot}$)
- Schwarzschildradius jetzt ≈ 1 AU

IAAT

Computer-Simulation des Proton-Proton-Stoßes am LHC.

Spekulationen über Mikro Schwarze Löcher




The Lord did not create the World
in order to exclude himself
from certain parts of it...
Walter Greiner

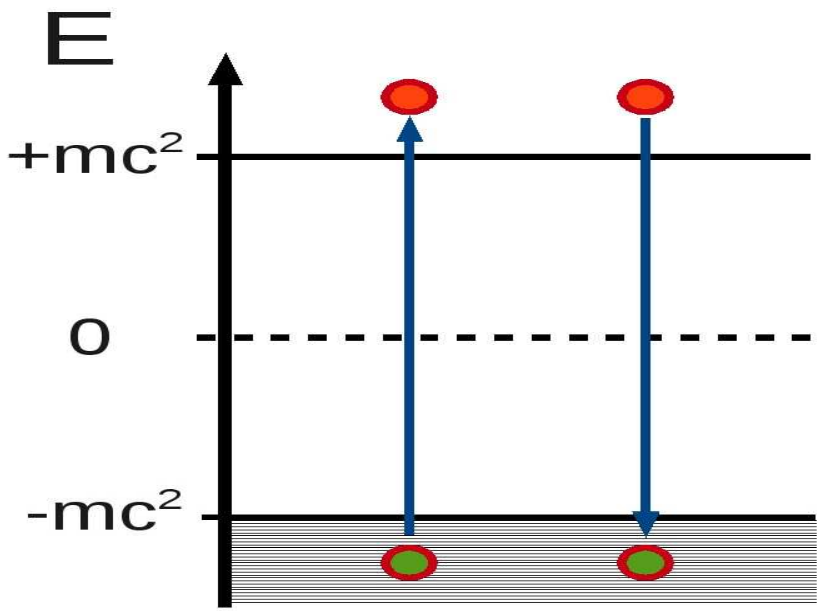
PSEUDO-COMPLEX GENERAL RELATIVITY

Peter O. Hess (ICN-UNAM and FIAS)
and
Walter Greiner (FIAS)

FROM KLEIN-GORDON TO DIRAC

$$E = \pm\sqrt{p^2c^2 + m^2c^4} \quad \rightarrow \quad \frac{E}{c} = \vec{\alpha}\cdot\vec{p} + \beta mc^2$$

$$\left(\beta\alpha_i = \gamma_i \quad , \quad \beta = \gamma_0\right)$$



The Dirac equation yields an explanation for the vacuum.

Historical Relativistic Equations

Remember the historical fact

$$\frac{E^2}{c^2} = \vec{p}^2 + m_0^2 c^2$$

Klein-Gordon Equation

$$\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{m_0^2 c^2}{\hbar^2}\right)\Psi$$

Dirac Equation

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[\frac{\hbar c}{i}\left(\hat{\alpha}_1\frac{\partial}{\partial x^1} + \hat{\alpha}_2\frac{\partial}{\partial x^2} + \hat{\alpha}_3\frac{\partial}{\partial x_3}\right) + m_0 c^2\hat{\beta}\right]\Psi$$

Dirac Matrices

Recall the Dirac Matrices

$$\hat{\alpha}_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \qquad \hat{\alpha}_2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$\hat{\alpha}_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \qquad \hat{\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Gamma-Matrices

And the Dirac Gamma-Matrices

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Dirac Equation

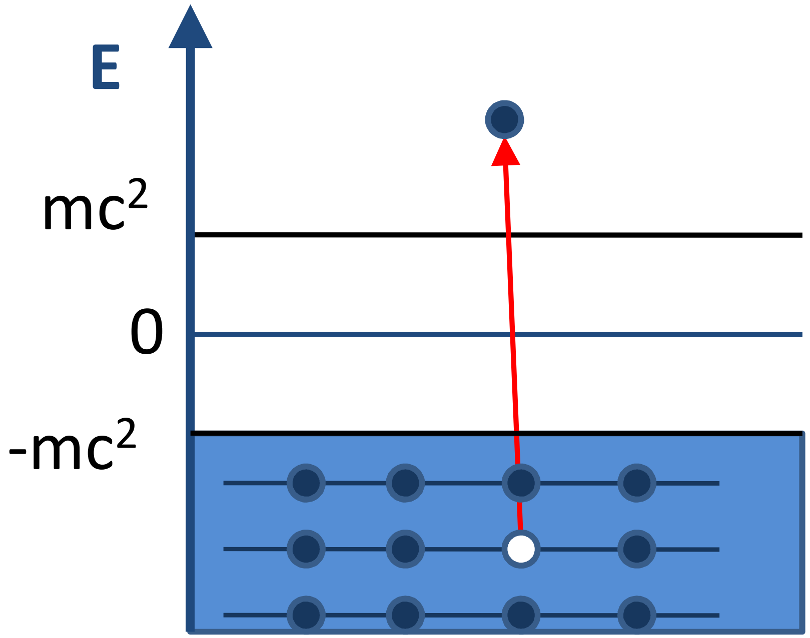
The Dirac Equation in a Covariant Form

$$i\hbar\left(\gamma^0\frac{\partial}{\partial x^0}+\gamma^1\frac{\partial}{\partial x^1}+\gamma^2\frac{\partial}{\partial x^2}+\gamma^3\frac{\partial}{\partial x^3}\right)\Psi = mc\Psi$$

From Klein-Gordon to Dirac Equation

Klein-Gordon Equation contains no spin

Dirac Equation describes particles spin 1/2



Dirac Sea

The Dirac equation predicts the existence of antiparticles and yields the model for the vacuum

CONTENT:

- Pseudo-complex variables
- Pseudo-complex General Relativity
- Schwarzschild metric as an example ("black holes", perihelion shift of Mercury)
- Pseudo-complex Robertson-Walker Metric
- Conclusions and outlook

FIRST ATTEMPTS.

- A. Einstein, Ann.Math. **46** (1945), 518.
- A. Einstein, Rev. Mod. Phys. **20** (1948), 35.

(Unification of gravitation and electrodynamics)

- C. Mantz, T. Prokopec, (2008); arXiv:0804.0213

(hermitian gravity and cosmology)

$$X^{\mu} = x^{\mu} + i\frac{l}{m}p^{\mu} \qquad i^2 = -1$$

(Introduction of the Planck length, l)

Born's equivalence principle: $\left[x^k, p^j\right] = i\hbar\delta_{ki}$ **but** $ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}$

(M. Born, *Proc. Roy. Soc.* A **165** (1938), 291 and
M. Born, *Rev. Mod. Phys.* **21** (1949), 463.)

Born's Reciprocity Theorem

Contrary to Einstein's General Relativity in Quantum Mechanics there is complete symmetry between coordinates and momenta

$$[x^i, p_j] = i\hbar\delta_{ij}$$

$$[x^i, x^j] = 0$$

$$[p_i, p_j] = 0$$

Thus suggests introducing the length element [Born, 1938]

$$dS^2 = g_{\mu\nu}(dx^\mu dx^\nu + \frac{l^2}{m^2} dp^\mu dp^\nu)$$

Lead by pure symmetry and dimensional arguments Born has introduced a scalar length parameter, which is unaffected by Lorentz transformations.

PROPOSAL
(M. BORN)

$$d\Omega^2 = dx_\mu dx^\mu + l^2 du_\mu du^\mu = dx_\mu dx^\mu \left(1 + l^2 \frac{du_\mu}{d\tau}\frac{du^\mu}{d\tau}\right)$$

$$\rightarrow dx_\mu dx^\mu \left(1 - l^2 a^2\right)$$

$$a^2 = -a_\mu a^\mu = \frac{du_\mu}{d\tau}\frac{du^\mu}{d\tau} \quad \Longrightarrow \quad l^2 \leq \frac{1}{a^2}$$

l is a minimal length

Maximal acceleration!!!
E.R. Caianiello (1981), H.E. Brandt, R.G. Beil (1980’s)
S.G. Low (1990’s and more recently: representation theory)

PSEUDO-COMPLEX VARIABLES

$$X = X_1 + IX_2 \quad , \quad I^2 = 1$$

Alternative: $\sigma_{\pm} = \frac{1}{2}(1 \pm I) \Rightarrow X = X_{+}\sigma_{+} + X_{-}\sigma_{-}$

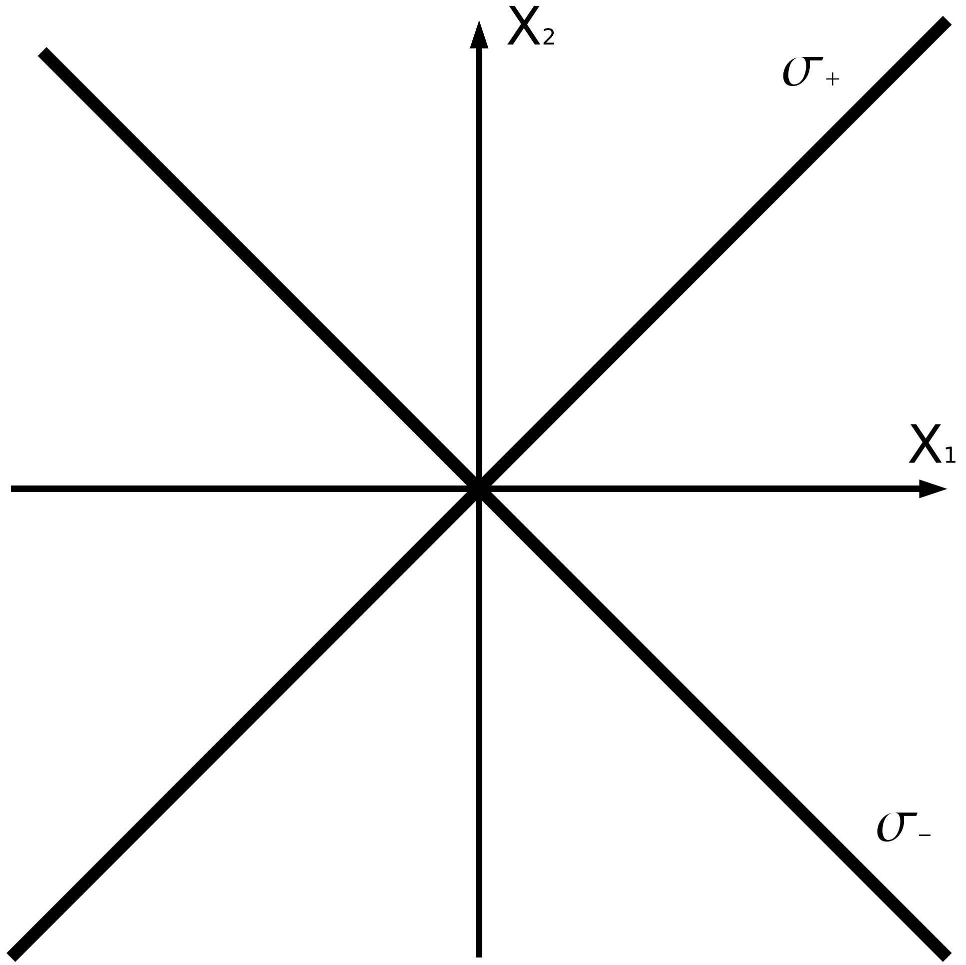
$$\sigma_{\pm}^2 = \sigma_{\pm} \qquad \sigma_{+}\sigma_{-} = 0$$

Permits the independent treatment of the two components!

Pseudo-Complex conjugate: $X^* = X_1 - IX_2 = X_{-}\sigma_{+} + X_{+}\sigma_{-} \quad (\sigma_{\pm} = \sigma_{\mp})$

Null-Norm : $X = X_{\pm}\sigma_{\pm} \Rightarrow |X|^2 = X^*X = 0\,!$

(Other names in the literature: Para-complex, hyperbolic, hypercomplex, semi-complex Variables).



Plane of the pseudo-complex variable X. Shown are the pseudo-real, pseudo-imaginary components and the zero divisor basis.

Rules for manipulation are similar as with real and complex numbers/variables:

$$F(X) = F(X_+)\sigma_+ + F(X_-)\sigma_-$$

$$F(X)G(X) = F(X_+)G(X_+)\sigma_+ + F(X_-)G(X_-)\sigma_-$$

$$\frac{F(X)}{G(X)} = \frac{F(X)G^*(X)}{G(X)G^*(X)} = \frac{F(X_+)}{G(X_+)}\sigma_+ + \frac{F(X_-)}{G(X_-)}\sigma_-$$

$$\frac{DF(X)}{DX} = \lim_{\Delta X \to 0} \frac{F(X + \Delta X) - F(X)}{\Delta X}$$

CONSEQUENCE OF THE PSEUDO-COMPLEX EXTENSION FOR THE LORENTZ SYMMETRY

Finite transformation: $e^{i\omega_{\mu\nu}\Lambda^{\mu\nu}}$ $\Lambda^{\mu\nu} =$ Normal Lorentz-transformation

$$\omega_{\mu\nu} = \omega^1_{\mu\nu} + I\omega^2_{\mu\nu} = \omega^+_{\mu\nu}\sigma_+ + \omega^-_{\mu\nu}\sigma_-$$

$$\omega^\pm_{\mu\nu} = \omega^1_{\mu\nu} \pm \omega^2_{\mu\nu}$$

→ $e^{i\omega_{\mu\nu}\Lambda^{\mu\nu}} = e^{i\omega^+_{\mu\nu}\Lambda^{\mu\nu}}\sigma_+ + e^{i\omega^-_{\mu\nu}\Lambda^{\mu\nu}}\sigma_- \quad \rightarrow \quad SO_+(3,1) \;\; SO_-(3,1) \quad (\sigma_+\sigma_- = 0!)$

Direct product of two Lorentz groups because both commute with each other. The normal Lorentz group is obtained for a vanishing pseudo-complex part in omega.

Pseudo-complex world line: $X^\mu = x^\mu + Ilu^\mu$

consequence: Appearance of a minimal Length "ℓ".

It is a scalar parameter!

Minimal Length = maximal acceleration

Remark:

Pseudo-complex world line: $$X^{\mu} = x^{\mu} + Ilu^{\mu}$$

x^{μ} **= position in 4-space**

u^{μ} **= basis vector of the tangent space at** x^{μ}

with units of the 4-velocity

NEW VARIATIONAL PRINCIPLE:

Define the action through the Lagrangian

$$S = \int L d\tau$$

for the variation we require

$$\delta S = \delta \int L d\tau \quad \in \quad Zero\ Divisor \quad \Rightarrow$$

$$\delta S = \xi \sigma_{-} \quad (convention)$$

this results in the equations of motion

$$\frac{D}{Ds}\left(\frac{DL}{DX^{\mu}}\right) - \left(\frac{DL}{DX^{\mu}}\right) \quad \in \quad Zero\ Divisor$$

(F. Schuller, PhD thesis, University of Cambridge (2003);
F. Schuller, *Ann. Phys. (N.Y.)* **299** (2002), 174,
F. S. has proposed this general variation principle in his thesis at Cambridge.)

Pseudo Complex Field Theory

Fields, variables and masses are pseudo complex

Scalar Field

$$L = \frac{1}{2}\left(D_\mu \Phi D^\mu \Phi - M^2 \Phi^2\right)$$

$$D_\mu = \frac{\partial}{\partial X^\mu}$$

Dirac Field

$$L = \overline{\Psi}\left(i\gamma^\mu D_\mu - M\right)\Psi$$

Field Propagators

Propagator for the scalar field

$$\frac{1}{p^2 - M_+^2} - \frac{1}{p^2 - M_-^2}$$

Propagator for the Dirac field

$$\frac{1}{\gamma_\mu p^\mu - M_+} - \frac{1}{\gamma_\mu p^\mu - M_-}$$

Regularization via Pauli-Villars is automatically included within the theory!

Can one extend the General Relativity to pseudo-complex coordinates?

EXTENSION OF THE THEORY OF GENERAL RELATIVITY:

- The metric is pseudo-complex, without torsion:

$$g_{\mu\nu} = g^{+}_{\mu\nu}\sigma_{+} + g^{-}_{\mu\nu}\sigma_{-} \quad , \quad g_{\mu\nu} = g_{\nu\mu}$$

* pseudo-complex length element

$$d\omega^2 = g_{\mu\nu} DX^{\mu} DX^{\nu}$$

$$= g^{+}_{\mu\nu} DX^{\mu}_{+} DX^{\nu}_{+} \sigma_{+} + g^{-}_{\mu\nu} DX^{\mu}_{-} DX^{\nu}_{-} \sigma_{-}$$

Parallel Transport and Christophel symbols:

$$
\begin{aligned}
D\xi^\mu &= \Gamma^\mu_{\nu\lambda} DX^\nu \xi^\lambda \\
&= \Gamma^{+}_{\nu\lambda}{}^{\mu} DX^\nu_+ \xi^\lambda_+ \sigma_+ + \Gamma^{-}_{\nu\lambda}{}^{\mu} DX^\nu_- \xi^\lambda_- \sigma_- \\
&= d\xi^\mu_+ \sigma_+ + d\xi^\mu_- \sigma_- \, ,
\end{aligned}
$$

$$
\begin{aligned}
\Gamma^\lambda_{\mu\nu} &= -\left\{ \begin{matrix} \lambda \\ \nu \quad \mu \end{matrix} \right\} \\
&= -\left\{ \begin{matrix} \lambda \\ \nu \quad \mu \end{matrix} \right\}_+ \sigma_+ - \left\{ \begin{matrix} \lambda \\ \nu \quad \mu \end{matrix} \right\}_- \sigma_- \, ,
\end{aligned}
$$

$$
\Gamma^{\pm}_{\mu\nu}{}^{\lambda} = -\left\{ \begin{matrix} \lambda \\ \nu \quad \mu \end{matrix} \right\}_\pm = -g^{\lambda\kappa} \left[\nu\mu, \kappa\right]_\pm \, .
$$

$$[\mu\nu, \kappa] = \frac{1}{2}\left(\frac{Dg_{\mu\kappa}}{DX^{\nu}} + \frac{Dg_{\nu\kappa}}{DX^{\mu}} - \frac{Dg_{\mu\nu}}{DX^{\kappa}}\right).$$

Covariant derivative of a contravariant 4-vector:

$$\begin{aligned}\xi^{\mu}_{||\nu} &= \xi^{\mu}_{|\nu} + \left\{ {\mu \atop \nu\ \lambda} \right\} \xi^{\lambda} \\ &= \left(\xi^{\mu}_{+|\nu} + \left\{ {\mu \atop \nu\ \lambda} \right\}_{+} \xi^{\lambda}_{+}\right)\sigma_{+} + \left(\xi^{\mu}_{-|\nu} + \left\{ {\mu \atop \nu\ \lambda} \right\}_{-} \xi^{\lambda}_{-}\right)\sigma_{-},\end{aligned}$$

The covariant derivative of the metric is zero.
This assures a universal metric

$$g^{\pm}_{\mu\nu|\lambda} - g^{\pm}_{\mu\kappa} \left\{ \begin{matrix} \kappa \\ \nu \quad \lambda \end{matrix} \right\}_{\pm} = [\mu\lambda, \nu]_{\pm} \, ,$$

$$\begin{aligned} g^{\pm}_{\mu\nu||\lambda} &= g^{\pm}_{\mu\nu|\lambda} - \left\{ \begin{matrix} \kappa \\ \nu \quad \lambda \end{matrix} \right\}_{\pm} g^{\pm}_{\mu\kappa} - \left\{ \begin{matrix} \kappa \\ \mu \quad \lambda \end{matrix} \right\}_{\pm} g^{\pm}_{\kappa\nu} \\ &= [\mu\lambda, \nu]_{\pm} - g^{\pm}_{\kappa\nu} \left\{ \begin{matrix} \kappa \\ \mu \quad \lambda \end{matrix} \right\}_{\pm} . \end{aligned}$$

$$g_{\mu\nu||\lambda} = g^{+}_{\mu\nu||\lambda}\sigma_{+} + g^{-}_{\mu\nu||\lambda}\sigma_{-} = 0 \, ,$$

or

$$g^{\pm}_{\mu\nu||\lambda} = 0 \, ,$$

COORDINATES:

$$X^{\pm}_{\mu} = g^{\pm}_{\mu\nu} X^{\nu}_{\pm}$$

$$X^{\mu}_{\pm} = g^{\mu\nu}_{\pm} X^{\pm}_{\nu}$$

$$x_{\mu} \pm l u_{\mu} = g^{\pm}_{\mu\nu}(x^{\nu} \pm l u^{\nu})$$

$$x^{\mu} \pm l u^{\mu} = g^{\mu\nu}_{\pm}(x_{\nu} \pm l u_{\nu})$$

$$\begin{aligned} x_{\mu} &= \frac{1}{2}(g^{+}_{\mu\nu} + g^{-}_{\mu\nu})x^{\nu} + l\frac{1}{2}(g^{+}_{\mu\nu} - g^{-}_{\mu\nu})u^{\nu} \\ &= g^{0}_{\mu\nu}x^{\nu} + l h_{\mu\nu}u^{\nu} \end{aligned}$$

$$\begin{aligned} l u_{\mu} &= \frac{1}{2}(g^{+}_{\mu\nu} - g^{-}_{\mu\nu})x^{\nu} + l\frac{1}{2}(g^{+}_{\mu\nu} + g^{-}_{\mu\nu})u^{\nu} \\ &= l g^{0}_{\mu\nu}u^{\nu} + h_{\mu\nu}x^{\nu} . \end{aligned}$$

THE LENGTH ELEMENT:

$$d\omega^2 = g^{+}_{\mu\nu} DX^{\mu}_{+} DX^{\nu}_{+} \sigma_{+} + g^{-}_{\mu\nu} DX^{\mu}_{-} DX^{\nu}_{-} \sigma_{-}$$

Condition of reality:

$$d\omega^{*2} = d\omega^2 .$$

→

$$l(dx_{\mu} du^{\mu} + du_{\mu} dx^{\mu}) = 0$$

→Dispersion relation! (will be used as a constriction)

(see next slide)

$$d\omega^2 = dx_{\mu} dx^{\mu} + l^2 du_{\mu} du^{\mu}$$

Limit of standard GR (pseudo-complex part=0): old results, where the dispersion relation is AUTOMATICALLY fulfilled.

4-velocity: $u_\mu = \frac{dx_\mu}{ds}$; $u^\mu = \frac{dx^\mu}{ds}$

Dispersion relation: $u_\mu u^\mu = const$

Differentiation:

$$d\left(u_\mu u^\mu\right) = 0 \quad \Rightarrow \quad du_\mu \frac{dx^\mu}{ds} + \frac{dx_\mu}{ds} du^\mu = 0$$

$$\Rightarrow \quad du_\mu dx^\mu + dx_\mu du^\mu = 0$$

THE LENGHT ELEMENT (CONT.)

$$d\omega^2 = g^0_{\mu\nu}(dx^\mu dx^\nu + l^2 du^\mu du^\nu) + l h_{\mu\nu}(dx^\mu du^\nu + du^\mu dx^\nu)\,.$$

$$g^R_{\mu\nu} = \frac{1}{2}\left(g^+_{\mu\nu} + g^-_{\mu\nu}\right) = g^0_{\mu\nu}$$

$$g^I_{\mu\nu} = \frac{1}{2}\left(g^+_{\mu\nu} - g^-_{\mu\nu}\right) = h_{\mu\nu}\,,$$

→ $$d\omega^2 \approx g^0_{\mu\nu} dx^\mu dx^\nu\,.$$

Problems:

$$g^0_{\mu\lambda} g_0^{\lambda\nu} + h_{\mu\lambda} h^{\lambda\nu} = \delta_{\mu\nu}$$

$$h_{\mu\lambda} g_0^{\lambda\nu} + g^0_{\mu\lambda} h^{\lambda\nu} = 0$$

The g- und h-"matrix", alone, are no tensors!

Equations for the matter free space

Setting $L = \sqrt{-g}R$ we get

Equations for the matter free space

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \quad \in \quad Zero\, Divisor$$

This suggests that the right hand side is related to the energy momentum tensor

EQUATIONS OF MOTION IN THE SCHWARZSCHILD PROBLEM

(1) $$\ddot{X}^{\mu} + \begin{Bmatrix} & \mu & \\ \nu & & \lambda \end{Bmatrix} \dot{X}^{\nu} \dot{X}^{\lambda} = \xi^{\mu} \sigma_{-} \quad \rightarrow$$

(2)

$$\ddot{X}^{0} + \nu' \dot{R} \dot{X}^{0} = \xi^{0} \sigma_{-}$$

$$\ddot{R} + \frac{1}{2} \lambda' \dot{R}^{2} + \frac{1}{2} \nu' e^{\nu - \lambda} \left(\dot{X}^{0} \right)^{2} - e^{-\lambda} R \dot{\vartheta}^{2} - R \sin^{2} \vartheta e^{-\lambda} \dot{\varphi}^{2} = \xi^{R} \sigma_{-}$$

$$\ddot{\vartheta} + \frac{2}{R} \dot{\vartheta} \dot{R} - \sin \vartheta \cos \vartheta \dot{\varphi}^{2} = \xi^{\vartheta} \sigma_{-}$$

$$\ddot{\varphi} + 2 \cot \vartheta \, \dot{\varphi} \dot{\vartheta} + \frac{2}{R} \dot{R} \dot{\varphi} = \xi^{\varphi} \sigma_{-}$$

Comparison of (1) with (2) → Christoffel symbols of the 2. kind

$$\left(\nu' = \frac{D\nu}{DR} \quad , \quad \dot{R} = \frac{DR}{Ds}, \qquad etc. \right)$$

CHRISTOFFEL SYMBOLS OF THE 2.KIND

$$\begin{Bmatrix} & 0 & \\ 1 & & 0 \end{Bmatrix} = \begin{Bmatrix} & 0 & \\ 0 & & 1 \end{Bmatrix} = \frac{1}{2}\nu'$$

$$\begin{Bmatrix} & 1 & \\ 0 & & 0 \end{Bmatrix} = \frac{1}{2}\nu' e^{\nu-\lambda}$$

$$\begin{Bmatrix} & 1 & \\ 1 & & 1 \end{Bmatrix} = \frac{1}{2}\lambda'$$

$$\begin{Bmatrix} & 1 & \\ 2 & & 2 \end{Bmatrix} = -\mathrm{Re}^{-\lambda}$$

$$\begin{Bmatrix} & 1 & \\ 3 & & 3 \end{Bmatrix} = -R\sin^2\vartheta e^{-\lambda}$$

$$\begin{Bmatrix} & 2 & \\ 2 & & 1 \end{Bmatrix} = \begin{Bmatrix} & 2 & \\ 1 & & 2 \end{Bmatrix} = \frac{1}{R}$$

$$\begin{Bmatrix} & 2 & \\ 3 & & 3 \end{Bmatrix} = -\sin\vartheta\cos\vartheta$$

$$\begin{Bmatrix} & 3 & \\ 2 & & 3 \end{Bmatrix} = \begin{Bmatrix} & 3 & \\ 3 & & 2 \end{Bmatrix} = \cot\vartheta$$

$$\begin{Bmatrix} & 3 & \\ 1 & & 3 \end{Bmatrix} = \begin{Bmatrix} & 3 & \\ 3 & & 1 \end{Bmatrix} = \frac{1}{R}$$

THE RICCI-TENSOR:

$$\mathcal{R}_{\mu\nu} = \left\{ {\beta \atop \beta \quad \nu} \right\}_{|\mu} - \left\{ {\beta \atop \mu \quad \nu} \right\}_{|\beta} + \left\{ {\beta \atop \tau \quad \mu} \right\} \left\{ {\tau \atop \beta \quad \nu} \right\} - \left\{ {\beta \atop \tau \quad \beta} \right\} \left\{ {\tau \atop \mu \quad \nu} \right\} .$$

→

$$\mathcal{R}_{00} = -\frac{e^{\nu-\lambda}}{2}\left(\nu'' + \frac{\nu'^2}{2} - \frac{\lambda'\nu'}{2} + \frac{2\nu'}{R}\right)$$

$$\mathcal{R}_{11} = \frac{1}{2}\left(\nu'' + \frac{\nu'^2}{2} - \frac{\lambda'\nu'}{2} - \frac{2\lambda'}{R}\right)$$

$$\mathcal{R}_{22} = (e^{-\lambda}R)' - 1$$

$$\mathcal{R}_{33} = \sin^2\theta[(e^{-\lambda}R)' - 1] .$$

SOLUTION:

$$d\omega^2 = e^{\nu}(DX^0)^2 - e^{\lambda}(DR)^2 - R^2\left[(D\vartheta)^2 + \sin^2\vartheta(D\varphi)^2\right]$$

$$\nu' + \lambda' = \frac{1}{2}R_-\left(\xi_0(R_-) - \xi_1(R_-)\right)\sigma_-$$

$$e^{-\lambda} = e^{\nu_+}\sigma_+ + e^{\nu_- - \int [R_-(\xi_0 - \xi_1)/2] DR_-}\sigma_-$$

The integral in the metric should vanish at large distances. In order to achieve this, the simplest way is to make the following assumption:

Assumption: $\Rightarrow e^{-\lambda} = e^{\nu}$

$$e^{-\lambda} = e^{\nu_+}\sigma_+ + e^{\nu_-}\sigma_- = e^{\nu}$$

SOLUTION:

$\sigma_+ - component:$

$$e^{-\lambda_+} = 1 - \frac{2M_+}{R_+}$$

$\sigma_- - component:$

$$e^{-\lambda_-} = 1 - \frac{2M_-}{R_-} + \frac{1}{R_-}\int \xi_2(R_-)dR_- .$$

definition:

$$\Omega = \int \xi_2 dR_- .$$

In order to approach the Schwarzschild solution for large distances, applying the before mentioned reality condition on the length element, leads to the identification of:

$$M_\pm = m$$

FURTHER PROBLEMS:

Ω=?

Ω should fall off with distance sufficiently, such that it does not contradict current measurements (solar system experiments). This leads to the minimal ansatz:

$$\Omega = \frac{B}{R_-^2} = b\frac{m^3}{R_-^2}$$

But it also can be:

$$\Omega = \frac{B}{R_-^k} = b\frac{m^{k+1}}{R_-^k}$$

Using: $\left(l << 1, \qquad R_\pm = r \pm l\dot{r} \approx r\right)$

Also possible:

$$\Omega = b\frac{m^3}{r^2}\exp\left\{-\alpha\left[\left(\frac{r}{2\mathrm{m}}\right)^k - 1\right]\right\}$$

here the contribution is arbitrary small for r>2m

The spherically symmetric Schwarzschild Solution

Equations for the matter free space

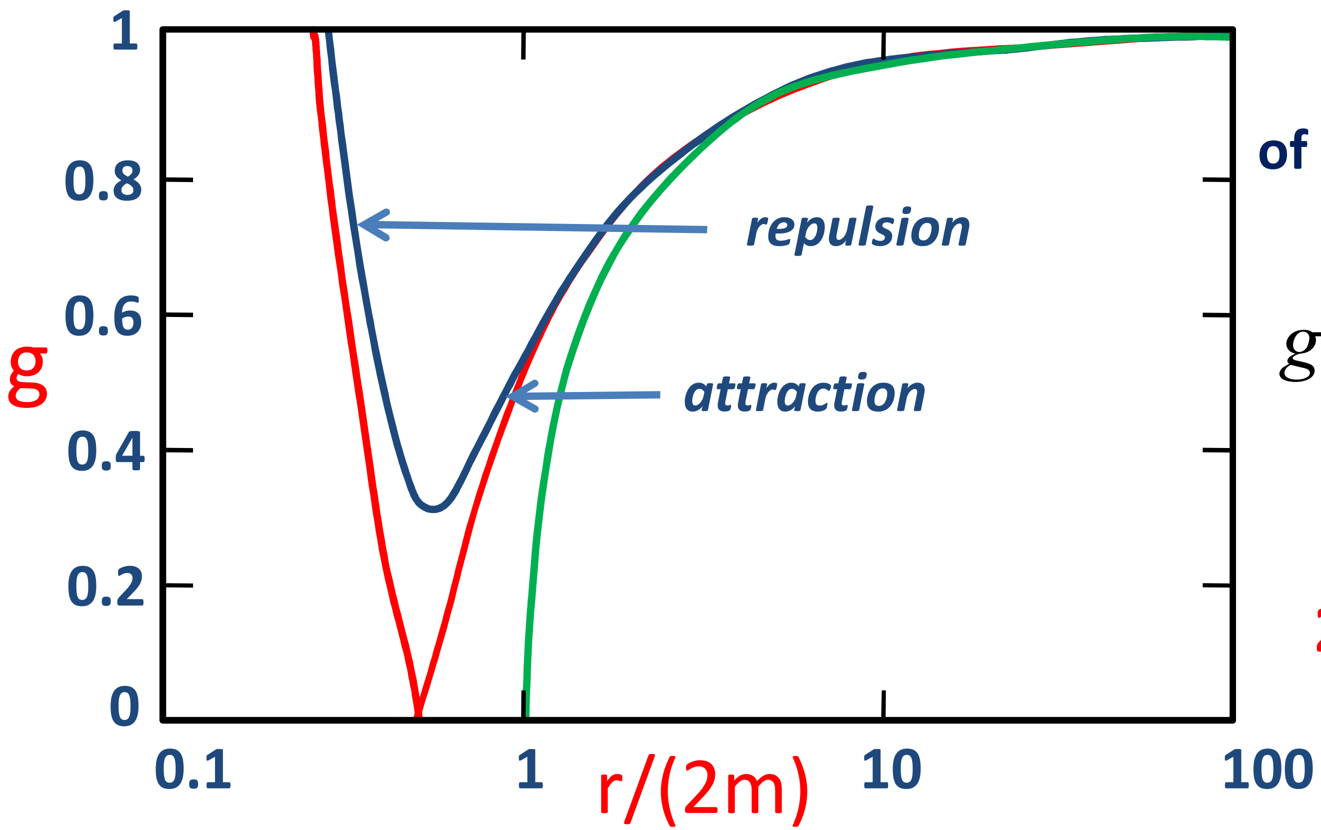
$$\mathbf{R}_{\mu\nu} = 0 \qquad \& \qquad \mathbf{R} = \mathbf{0}$$

We obtain the isotropic Schwarzschild solution

$$d\omega^2 \approx \left(1 - \frac{2m}{r} + \frac{B}{2r^2}\right)dt^2 - \left(\frac{1 - \frac{2m}{r} + \frac{B}{2r^2}}{\left(1 - \frac{2m}{r}\right)\left(1 - \frac{2m}{r} + \frac{B}{r^2}\right)}\right)dr^2$$

The Red Shift

The Red Shift g factor: $$d\tau \approx \sqrt{g_{00}^{0}}\,dt = \sqrt{\left(1 - \frac{2m}{r} + \frac{B}{2r^3}\right)}dt \equiv gdt$$



The metric component of the time must be positive

$$g_{00}^{0} > 0 \Rightarrow B > \frac{64}{27}m^3$$

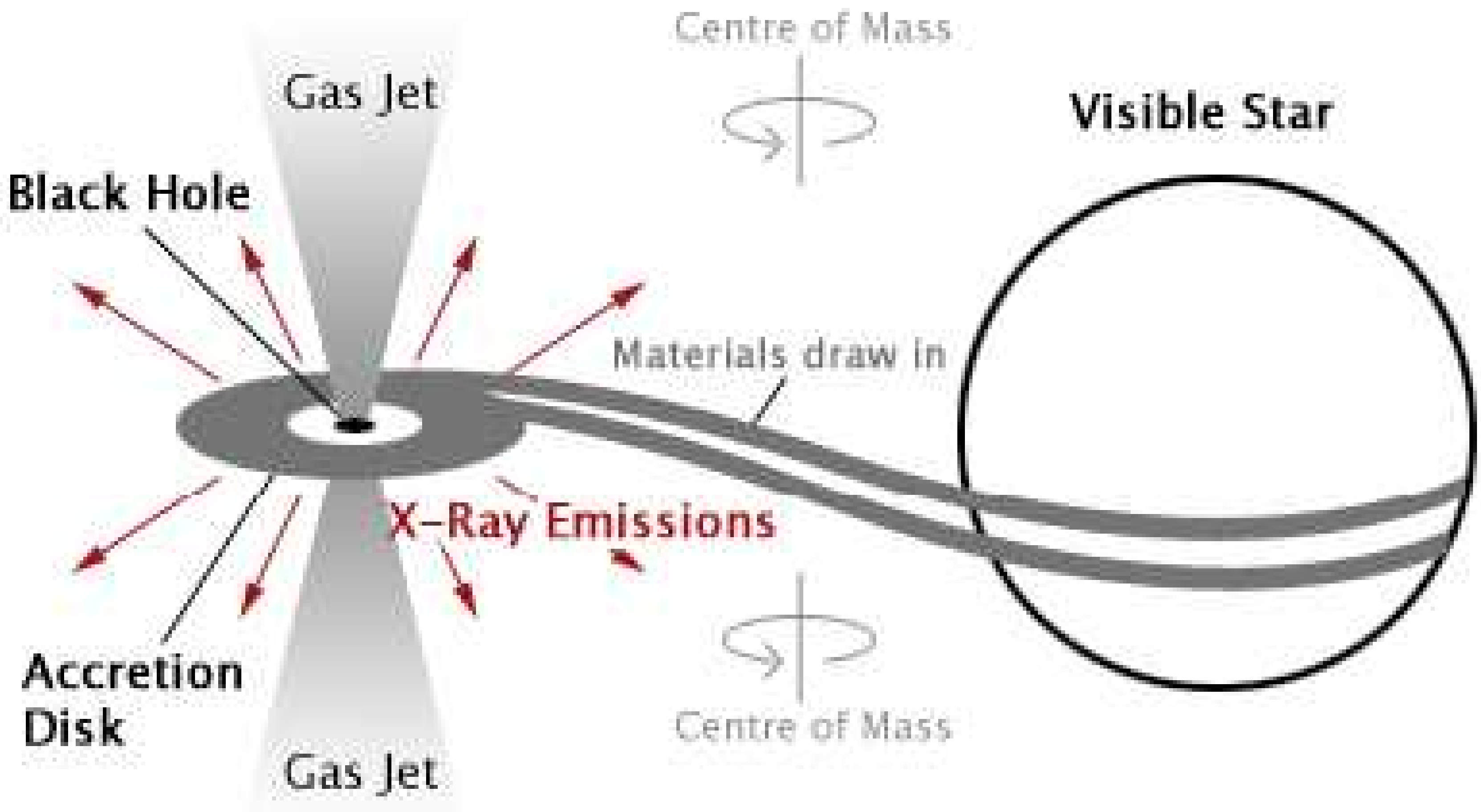
Antigravitation below 2/3 of the Schwarzschild radius!

Astronaut

A fatal fall into the black hole (tidal forces)

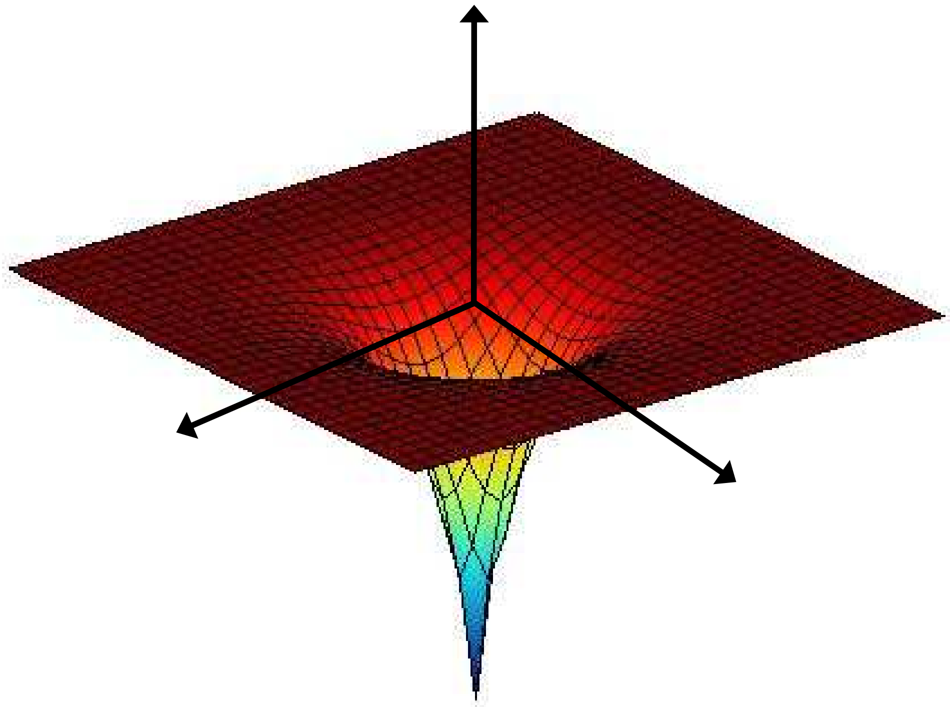
Binary Star

Binary Star with one Visible and one Black Hole Component

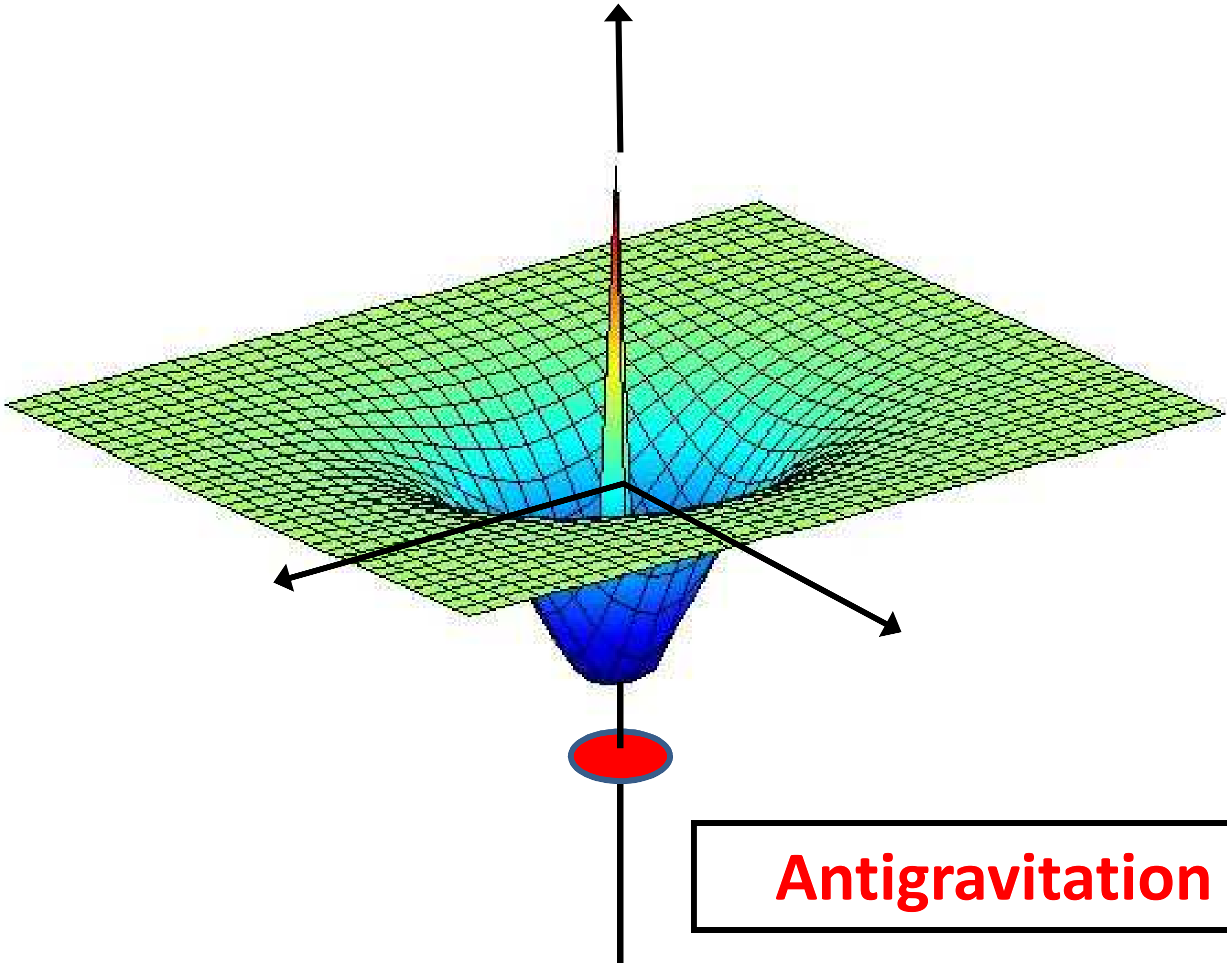


Typical Black Hole

Schwarzschild Solution embedded into the Euclidean Space

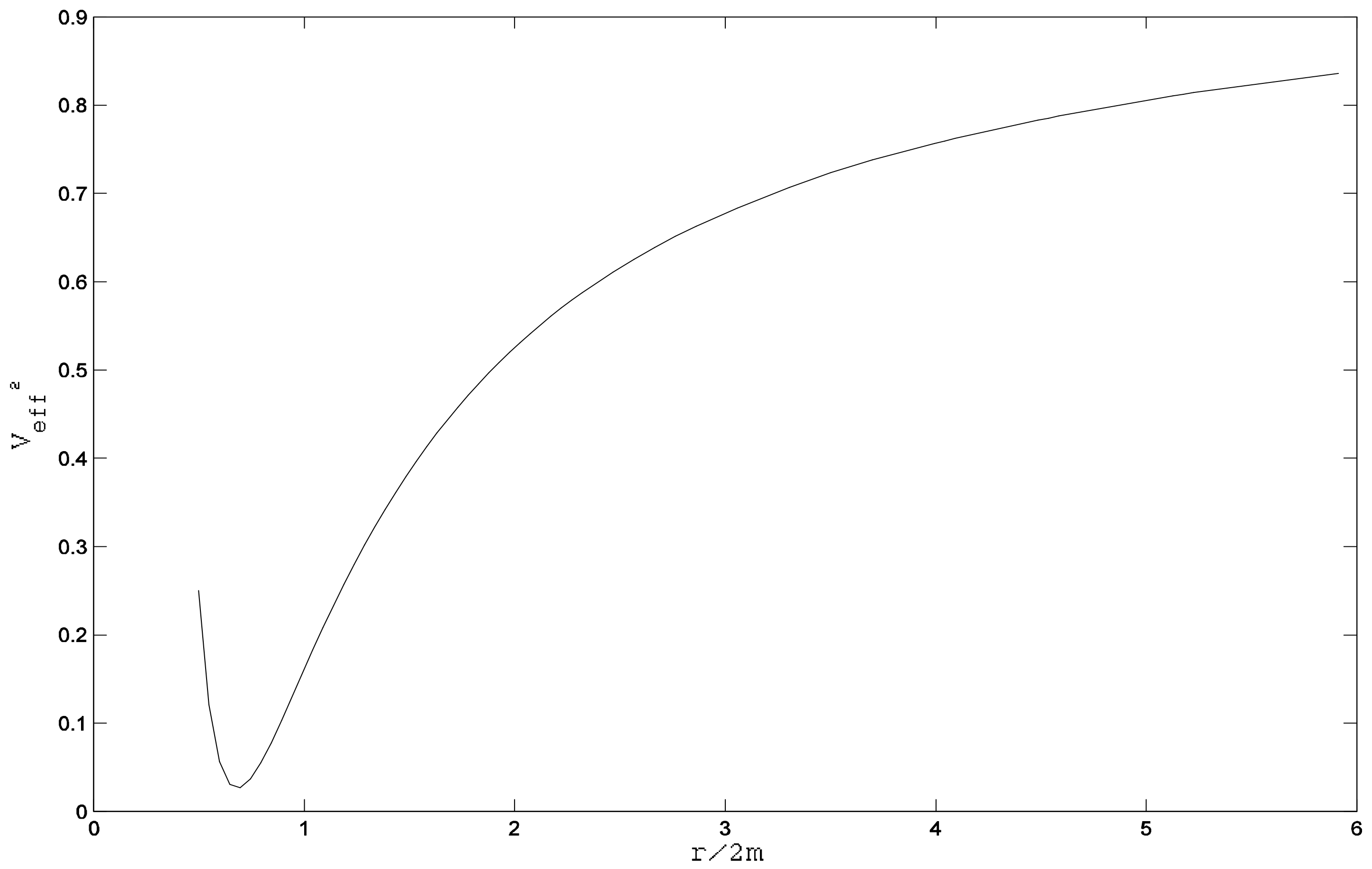


There are no Black Holes!



Antigravitation in the center

EFFECTIVE POTENTIAL
(MISNER ET AL., GRAVITATION)



(B=5m^3)

PARAMETRIZED-POST-NEWTON (PPN)-FORMALISM: ISOTROPIC COORDINATES

$$d\omega^2 = g^0_{00}(dX^0)^2 + g^0_{rr}(dr)^2 - r^2 d\Omega^2 = g^0_{00}(dX^0)^2 + f^2\left[d\bar{r}^2 - \bar{r}^2 d\Omega^2\right]$$

$$\bar{r} = \exp\left\{\int \frac{\sqrt{g_{rr}}}{r} dr\right\}$$

$$r^2 d\Omega = f^2 \bar{r}^2 d\Omega \qquad g^0_{rr} dr^2 = f^2 d\bar{r}^2 \quad \Rightarrow \quad f^2 = \frac{r^2}{\bar{r}^2}$$

$$\rightarrow \quad d\omega^2 \approx \left[1 - 2\left(\frac{m}{\bar{r}}\right) + 2\left(\frac{m}{\bar{r}}\right)^2 - \frac{3}{2}\left(1 - \frac{b}{3}\right)\left(\frac{m}{\bar{r}}\right)^3\right] dt^2$$

$$- \left[1 + 2\left(\frac{m}{\bar{r}}\right) + \frac{3}{2}\left(\frac{m}{\bar{r}}\right)^2\right]\left[d\bar{r}^2 + \bar{r}^2 d\Omega^2\right]$$

The correction appears only in the time element-

→ Perihelion shift, everything else is identical to standard GR.

(approx. 10^{-7}) in the perihelion shift

PSEUDO-COMPLEX ROBERTSON-WALKER UNIVERSE

$$d\omega^2 = (dX^0)^2 - e^{G(X^0,R)}\left(dR^2 + R^2 d\theta^2 + R^2 sin^2\theta d\phi^2\right)$$
$$= (dX^0)^2 - e^{G(X^0,R)} d\Sigma^2 \quad , \qquad (22)$$

with

$$G(X^0, R_1) = g(X^0) + f(R_1) \quad .$$

EQUATIONS OF MOTION

$$0 \quad \delta \int \left[(\dot{X}^0)^2 - e^G \left(\dot{R}^2 + R^2 \dot{\theta}^2 + R^2 sin^2 \theta \dot{\phi}^2 \right) \right] ds \;\; \epsilon \;\; \boldsymbol{P}^0 \quad ,$$

→

$$\begin{aligned}
\ddot{X}^0 + \frac{1}{2} g' e^G \left(\dot{R}^2 + R^2 \dot{\theta}^2 + R^2 sin^2 \theta \dot{\phi}^2 \right) &= \xi_0 \sigma_- \\
\ddot{R} + \frac{1}{2} f' \dot{R}^2 + g' \dot{X}^0 \dot{R} & \\
- \left(\frac{1}{2} f' + \frac{1}{R} \right) \left(\dot{R}^2 + R^2 \dot{\theta}^2 + R^2 sin^2 \theta \dot{\phi}^2 \right) &= \xi_R \sigma_- \\
\ddot{\theta} + 2 \left(\frac{1}{2} f' + \frac{1}{R} \right) \dot{R} \dot{\theta} + g' \dot{X}^0 \dot{\theta} - sin\theta cos\theta \dot{\phi}^2 &= \xi_\theta \sigma_- \\
\ddot{\phi} + 2 \left(\frac{1}{2} f' + \frac{1}{R} \right) + g' \dot{X}^0 \dot{\phi} + 2 cot\theta \dot{\theta} \dot{\phi} &= \xi_\phi \sigma_- (27)
\end{aligned}$$

Christophel symbols:

$$\left\{ {0 \atop 1\;\;1} \right\} = \frac{1}{2} g' e^G$$

$$\left\{ {0 \atop 2\;\;2} \right\} = \frac{1}{2} g' e^G R^2$$

$$\left\{ {0 \atop 3\;\;3} \right\} = \frac{1}{2} g' e^G R^2 sin^2\theta$$

$$\left\{ {1 \atop 0\;\;1} \right\} = \frac{1}{2} g'$$

$$\left\{ {1 \atop 1\;\;1} \right\} = \frac{1}{2} f'$$

$$\left\{ {1 \atop 2\;\;2} \right\} = -R^2 \left(\frac{1}{2} f' + \frac{1}{R} \right)$$

$$\left\{ {1 \atop 3\;\;3} \right\} = -R^2 \left(\frac{1}{2} f' + \frac{1}{R} \right) sin^2\theta$$

$$\left\{ {2 \atop 0\;\;2} \right\} = \frac{1}{2} g' = \left\{ {3 \atop 0\;\;3} \right\}$$

$$\left\{ {2 \atop 1\;\;2} \right\} = \left(\frac{1}{2} f' + \frac{1}{R} \right) = \left\{ {3 \atop 1\;\;3} \right\}$$

$$\left\{ {2 \atop 3\;\;3} \right\} = -sin\theta cos\theta$$

$$\left\{ {3 \atop 2\;\;3} \right\} = cot\theta \quad .$$

EQUATIONS OF MOTION-2:

$$
\begin{aligned}
\mathcal{R}_0^0 &= \frac{3}{2}g'' + \frac{3}{4}g'^2 \\
\mathcal{R}_1^1 &= \left(\frac{1}{2}g'' + \frac{3}{4}g'^2\right) - e^{-G}\left(f'' + \frac{f'}{R}\right) \\
\mathcal{R}_2^2 = \mathcal{R}_3^3 &= \left(\frac{1}{2}g'' + \frac{3}{4}g'^2\right) \\
&\quad - e^{-G}\left(\frac{1}{2}f'' + \frac{1}{4}f'^2 + \frac{3f'}{2R}\right) \quad .
\end{aligned}
$$

(3

Metric:

$$
g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -e^G & 0 & 0 \\ 0 & 0 & -e^G R^2 & 0 \\ 0 & 0 & 0 & -e^G R^2 sin^2\theta \end{pmatrix}
$$

$$-\frac{8\pi\kappa}{c^2}T_0^0 = \left[e^{-G}\left(f'' + \frac{f'^2}{4} + \frac{2f'}{R}\right) - \frac{3}{4}g'^2\right] + \xi_0\sigma_-$$

$$-\frac{8\pi\kappa}{c^2}T_1^1 = \left[e^{-G}\left(\frac{f'^2}{4} + \frac{f'}{R}\right) - g'' - \frac{3}{4}g'^2\right] + \xi_1\sigma_-$$

$$-\frac{8\pi\kappa}{c^2}T_2^2 = \left[e^{-G}\left(\frac{f''}{2} + \frac{f'}{2R}\right) - g'' - \frac{3}{4}g'^2\right] + \xi_2\sigma_-$$

$$-\frac{8\pi\kappa}{c^2}T_3^3 = \left[e^{-G}\left(\frac{f''}{2} + \frac{f'}{2R}\right) - g'' - \frac{3}{4}g'^2\right] + \xi_3\sigma_-$$

$$-\frac{8\pi\kappa}{c^2}T_\nu^\mu = 0 \quad , \quad \mu \neq \nu \quad . \qquad (38)$$

Homogeneity: $T_1^1 = T_2^2 = T_3^3 \quad , \quad \xi_1 = \xi_2 = \xi_3$

Dark energy density: $\rho_\Lambda = \frac{c^2}{8\pi\kappa}\xi_0\sigma_-$

(Pseudo-complex energy density)

PSEUDO-COMPLEX PRESSURE

Shifting ξ to the left hand side:

$$-\frac{8\pi\kappa}{c^2}\left(T_k^k + \frac{c^2}{8\pi\kappa}\xi_k\sigma_-\right) \quad \rightarrow \quad -\frac{8\pi\kappa}{c^2}\left(-\frac{p}{c^2} - \frac{p_\xi}{c^2}\right)$$

with

$$p_\xi = -\frac{c^4}{8\pi\kappa}\xi_k\sigma_-$$

negative pressure!

$$d\omega^2 = (dX^0)^2 - e^{g(X^0)} \frac{1}{\left(1 + \frac{kR^2}{4R_0^2}\right)^2} d\Sigma^2 \quad . \qquad (43)$$

$$\begin{aligned} e^{G(X^0,R)} &= \frac{\boldsymbol{R}(X^0)^2}{R_0^2 \left(1 + kR^2/(4R_0^2)\right)^2} \\ e^{g(X^0)} &= \boldsymbol{R}(t)^2 \\ e^{f(R)} &= \frac{1}{R_0^2 \left(1 + kR^2/(4R_0^2)\right)^2} \quad , \end{aligned}$$

$$(T^\mu_\nu) = \begin{pmatrix} \rho & & & \\ & -\frac{p}{c^2} & & \\ & & -\frac{p}{c^2} & \\ & & & -\frac{p}{c^2} \end{pmatrix} \quad , \qquad ($$

→

$$\frac{8\pi\kappa}{c^2}\rho = -\xi_0\sigma_- + \left[\frac{3k}{R(t)^2} + \frac{3}{c^2}\frac{R'(t)^2}{R(t)^2}\right]$$

$$\frac{8\pi\kappa}{c^2}\frac{p}{c^2} = \xi_1\sigma_- - \left[\frac{k}{R(t)^2} + \frac{R'(t)^2}{c^2 R(t)^2} + \frac{2R''(t)}{c^2 R(t)}\right]$$

k=0 → flat universe

$$\frac{4\pi\kappa}{c^2}\left(\rho + \frac{3p}{c^2}\right) = \frac{1}{2}(3\xi_1 - \xi_0)\sigma_- - \frac{3R''}{c^2 R}$$

$$\frac{4\pi\kappa}{c^2}\left(\rho + \frac{p}{c^2}\right) = \frac{1}{2}(\xi_1 - \xi_0)\sigma_- - \frac{R'^2 - RR''}{c^2 R^2}$$

ESTIMATION OF ξ_k -FUNCTIONS:

$$\xi_0 = \frac{3R''}{c^2 R}$$

$$\xi_1 = \frac{R'^2}{c^2 R^2} + \frac{2R''}{c^2 R}$$

Using the approximations:

$$\frac{R''_r}{R_r} = \frac{R''_r}{R'_r}\frac{R'_r}{R_r} = \left[\ln(R'_r)\right]' \frac{R'_r}{R_r} = \left[\ln H + \ln R_r\right]' \frac{R'_r}{R_r} = \left[\frac{H'}{H} + H\right] H = H' + H \approx H$$

$$R_\pm = r \pm l\dot{r} \approx r = R_r$$ **=radius of the universe**

we obtain: $\xi_0 \approx \frac{3}{c^2}\mathrm{H}^2 \approx \xi_1$. (k=0)

→ $\xi_1 = \beta\xi_0 + \left(\gamma\xi_0^2 + \delta\xi_0^3 + \ldots\right)$ (Ansatz)

LOCAL ENERGY CONSERVATION

$$\frac{d}{dt}(\rho R^3) + \frac{p}{c^2}\frac{dR^3}{dt} = \frac{c^2}{8\pi\kappa}\left[\frac{dR^3}{dt}(\xi_1 - \xi_0) - R^3\frac{d\xi_0}{dt}\right]\sigma$$

Condition: $\frac{d}{dt}$**(Energy + energy due to pressure) = 0**

$$\frac{d\xi_0}{dt} = \frac{d(lnR_-^3)}{dt}(\xi_1 - \xi_0) \quad .$$

Solution for ξ: assumption $\xi_1 = \beta\xi_0$ → $\xi_0 = \Lambda R_-^{3(\beta-1)}$

Solution for ρ: p=αρ **(α=0 →dust, =1/3 → radiation)**

$$\rho = \rho_0 R_r^{-3\left(1+\frac{\alpha}{c^2}\right)}$$

ACCELERATION:

Defining a rescaling, for notational reasons:

$$\widetilde{A} = \frac{c^2}{16\pi k} \frac{A}{\rho_0}$$

we obtain for the accelereation (rescaled):

$$\widetilde{R}_r'' = \frac{3R_r''}{4\pi k}\rho_0 = \widetilde{A}(3\beta - 1)R_r^{3(\beta-1)+1} - \left(1 + \frac{\alpha}{c^2}\right)R_r^{-3\left(1+\alpha/c^2\right)+1}$$

Contribution to dark energy yields acceleration if β>(1/3)

classical part yields de-acceleration

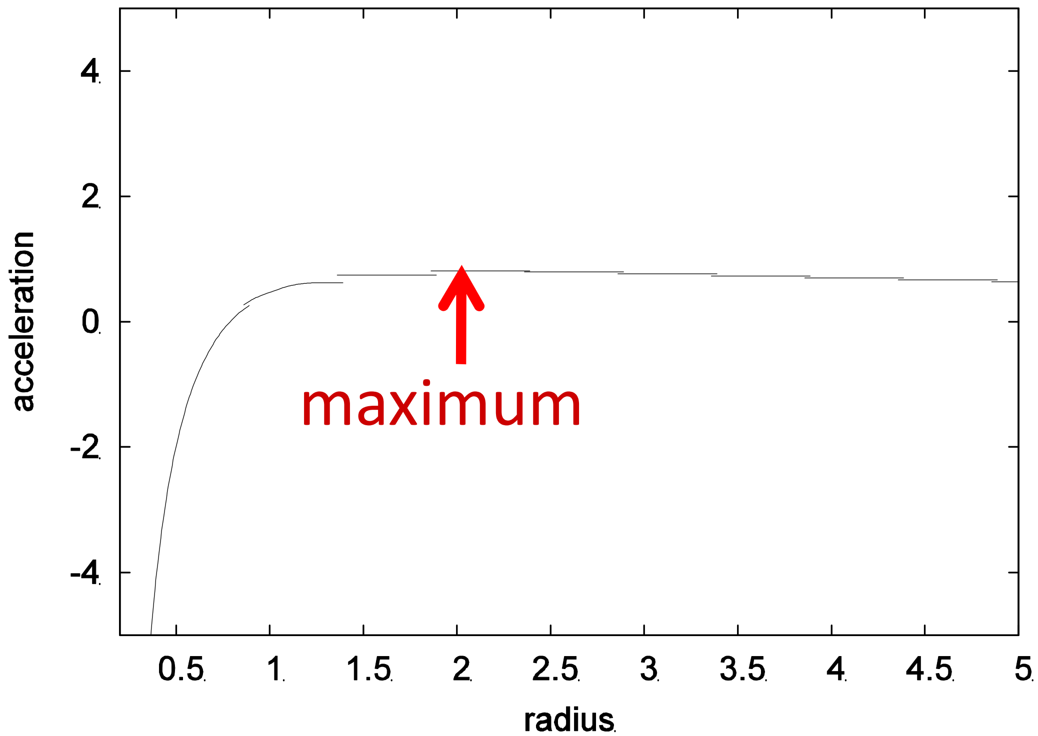
DIFFERENT CASES CASES:

- **β <1/3 → de-acceleration**

2) 2/3> β >1/3 → acceleration, with diminishing strength in R

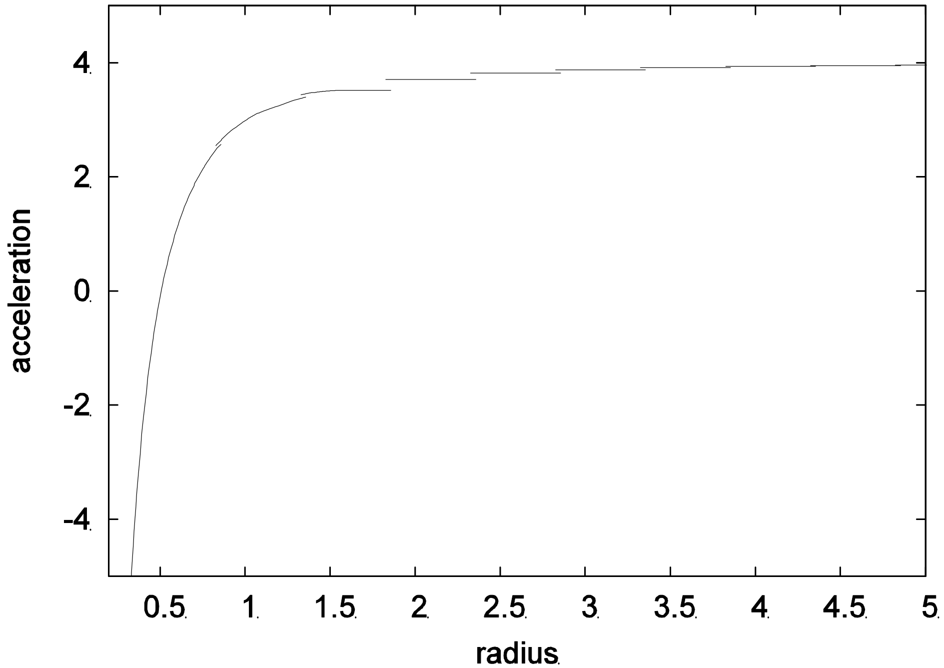
3)β >2/3 →acceleration, with increasing strength in R

β =1/2 , A=3



1. phase: de-acceleration
2. phase: increasing acceleration, until a maximum is reached
3. phase: decreasing acceleration

β =2/3 , A=4



No maximum!

1. phase: de-acceleration
2. phase: increasing acceleration, which for R→infinity approaches a constant value

In the standard GR, with a constant Λ, the asymptotic behavior is linear in R.

ESTIMATION FOR β : RELATION OF TO THE HUBBLE CONSTANT H AND ITS DERIVATIVE

$$\xi_0 = \frac{3R''}{c^2 R}$$

$$\xi_1 = \frac{R'^2}{c^2 R^2} + \frac{2R''}{c^2 R}$$

$$\frac{R_r''}{R_r} = \frac{R_r''}{R_r'}\frac{R_r'}{R_r} = \left[\ln(R_r')\right]'\frac{R_r'}{R_r} = \left[\ln H + \ln R_r\right]'\frac{R_r'}{R_r}$$

$$= \left[\frac{H'}{H} + H\right]H = H' + H$$

$$\Rightarrow \quad \xi_1 = \xi_0 + \frac{2}{c^2}H' = \beta\xi_0 = \beta\frac{3}{c^2}H^2$$

$$\Rightarrow \quad \beta = 1 + \frac{2}{3}\frac{H'}{H^2}$$

UNIVERSE WITH DUST AND RADIATION:

(P. Peebles, Rev. Mod.**75** (2003), 559.

$$\frac{R_r'}{R_r} = H_0^2\left[\Omega_d(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda f(z)\right]$$

dust **radiation** **dark energy within our theory**

$$f(z) = (1+z)^{3(\beta-1)} = (1+z)^{3(1+w)} \quad \Rightarrow \quad w = \beta - 2$$

with: $$\frac{R_{r0}}{R_r} = (1+z)$$ **and z as the redshift variable**

CONCLUSIONS AND OUTLOOK

- Variables = pseudo-complex
- Modified variational principle → dark energy
- Schwarzschild solution: Dark energy accumulates towards smaller radial distances → Collaps is avoided
- Pseudo-complex Robertson-Walker- Universe: different, old and new solutions, Estimation of ξ_k as a function of the Hubble constant H and its derivative.
- If $H^{'} \neq 0$, then w must be different from -1!

OUTLOOK

- Application of the PPN-formalism
- Results only different to standard GR in the perihelion shift
- Determine perihelion shift of … the moon???... (difficult due to the influence of the sun)