Highlights in Heavy-Ion Physics – A Symposium in Honour of Nikola Cindro
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Relationships Between New Observables and Properties of Finite Nuclei and Nuclear Matter

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INTRODUCTION

How to relate various observables and physical quantities, in particular those that can be measured with those which are difficult to assess?

What are observables that contain information about neutron skin?

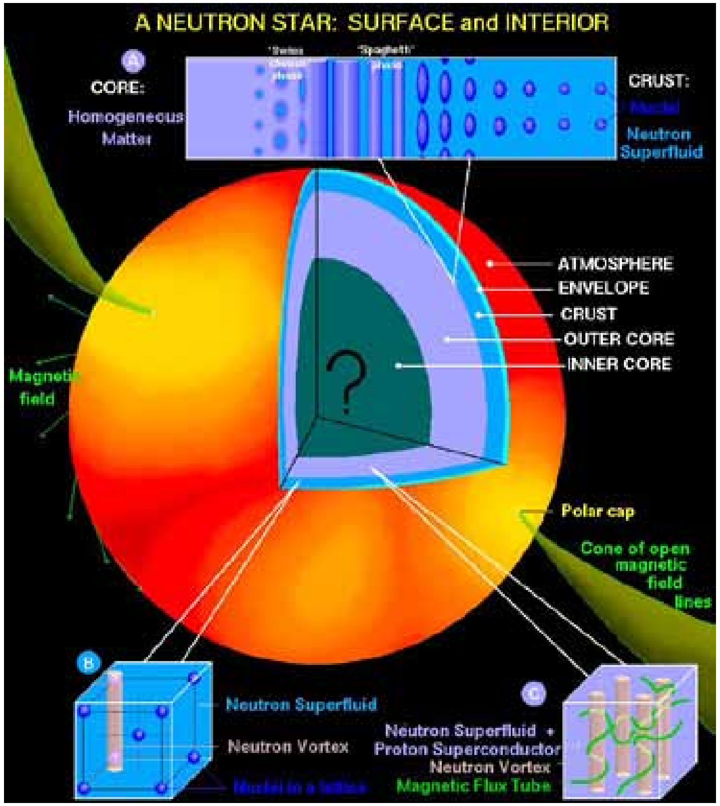
Connecting nuclear excitations with neutron radius of nuclei and nuclear matter properties

The role of E1 pygmy mode and dipole polarizability

Assessing statistical correlations between physical quantities

NEUTRON STARS AND SIZE OF NEUTRON SKIN

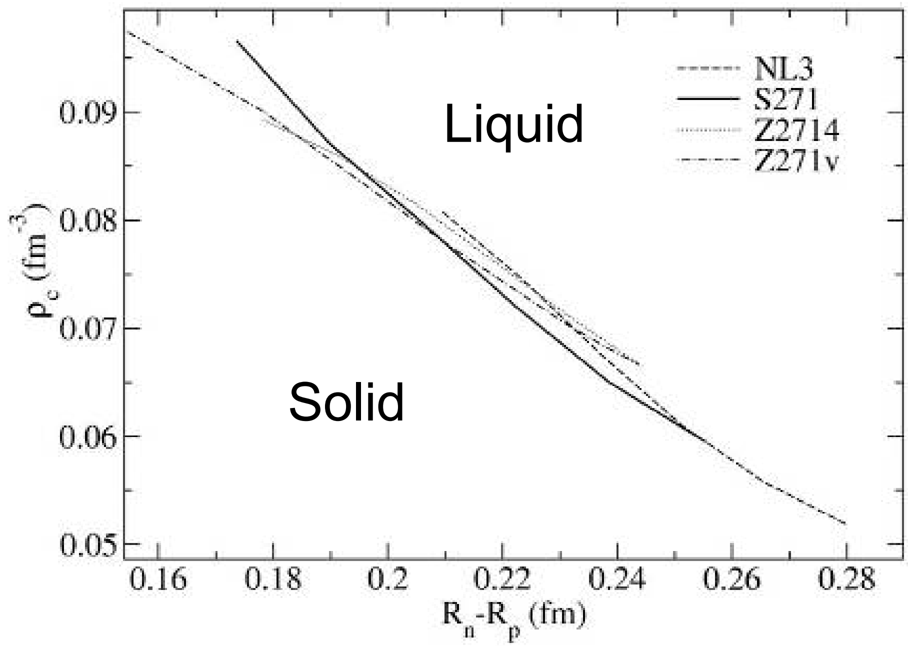
Neutron star structure and the size of neutron skin in ^{208}Pb



J.M. Lattimer & M. Prakash, Science 304 (2004) 536.

R_N calibrates equation of state (pressure vs. density) of Neutron Rich Matter

Implications for the structure of the crust of neutron stars



C.J. Horowitz, J. Piekarewicz, PRL 86, 5647 (2001).

MEASURING NUCLEON DISTRIBUTIONS IN NUCLEI

Nuclear charge densities have been accurately measured with elastic electron scattering

However, neutron densities in nuclei are probed mainly by hadron scattering, there are uncertainties in probe-nucleon interaction and reaction mechanism

Recent developments:

Lead (^{208}Pb) Radius Experiment : PREX

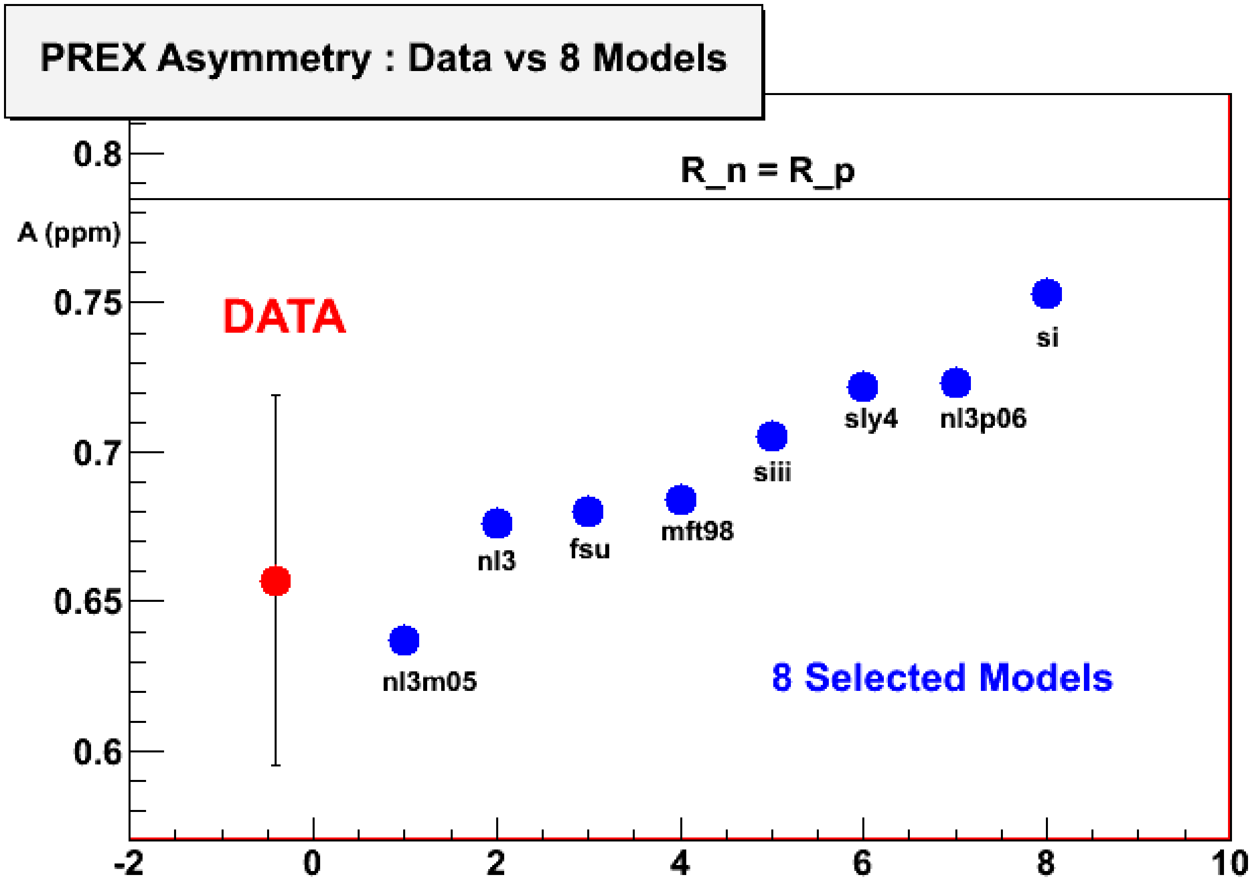
Elastic Scattering Parity Violating Asymmetry

E = 1 GeV, $\theta = 5^0$

electrons on lead

Adopted from R. Michaels, JLab

PREX PHYSICS INTERPRETATION



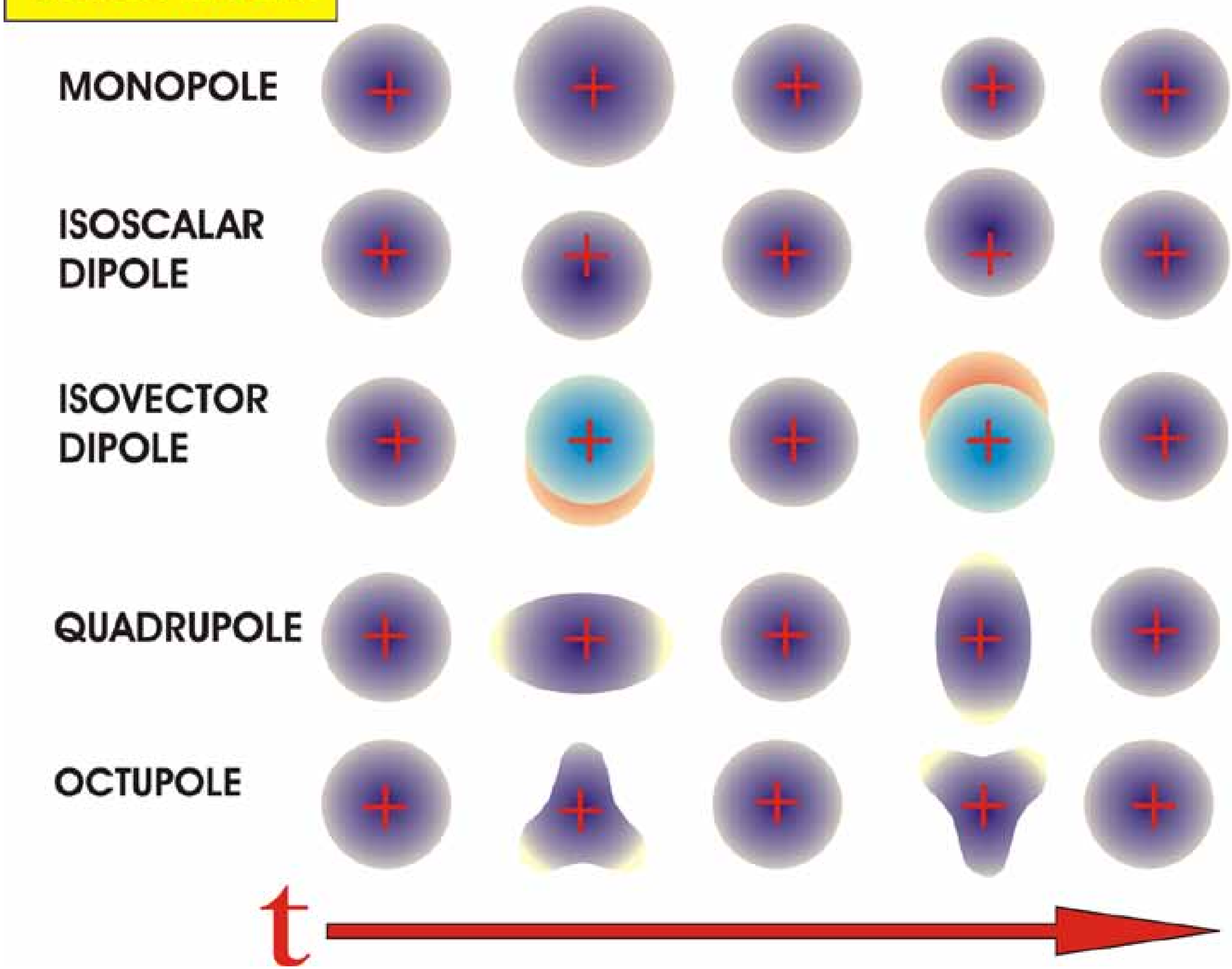
Preliminary:
Awaiting the “final” acceptance function

Neutron Skin = R_N - R_P = 0.31 + 0.15 - 0.17 fm

Adopted from R. Michaels, JLab

COLLECTIVE NUCLEAR MOTION AND NEUTRON SKIN

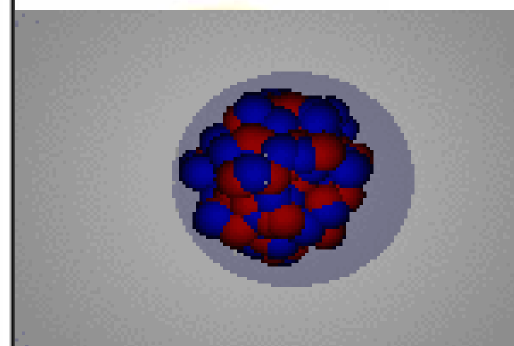
Stable nuclei



Exotic nuclei

WHAT IS EFFECT OF THE NEUTRON SKIN ON COLLECTIVE OSCILLATIONS ?

PYGMY MODE



OSCILLATIONS OF THE *NEUTRON RICH PERIPHERY* AGAINST THE *PROTON-NEUTRON CORE*

GIANT RESONANCES AND NEUTRON SKIN

Theoretically it has been shown that the cross section for GDR excitation depends strongly on the neutron-proton relative radius difference.

S. Shlomo et al., Phys. Rev. C 36, 1317 (1987)

G. R. Satchler, Nucl. Phys. A 472, 215 (1987)

K. Nakayama and G. Bertch, Phys. Rev. Lett. 59, 1053 (1987)

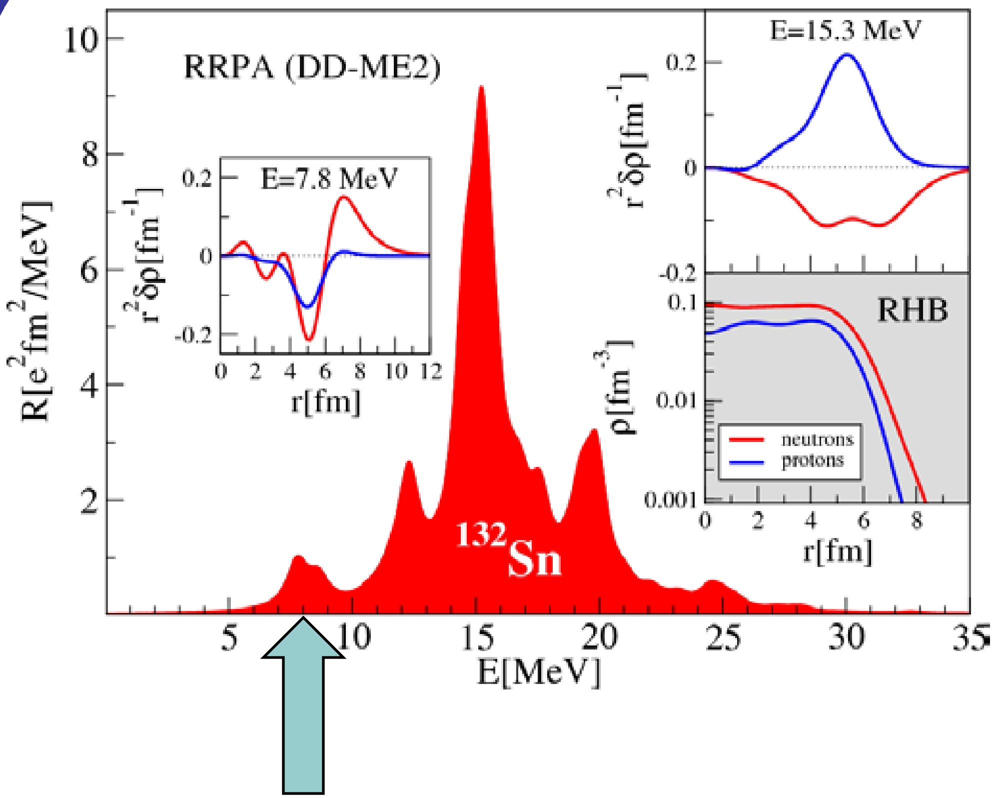
Krasznahorkay *et al. have used the excitation of the* giant dipole resonance to extract the neutron-skin thickness of nuclei.

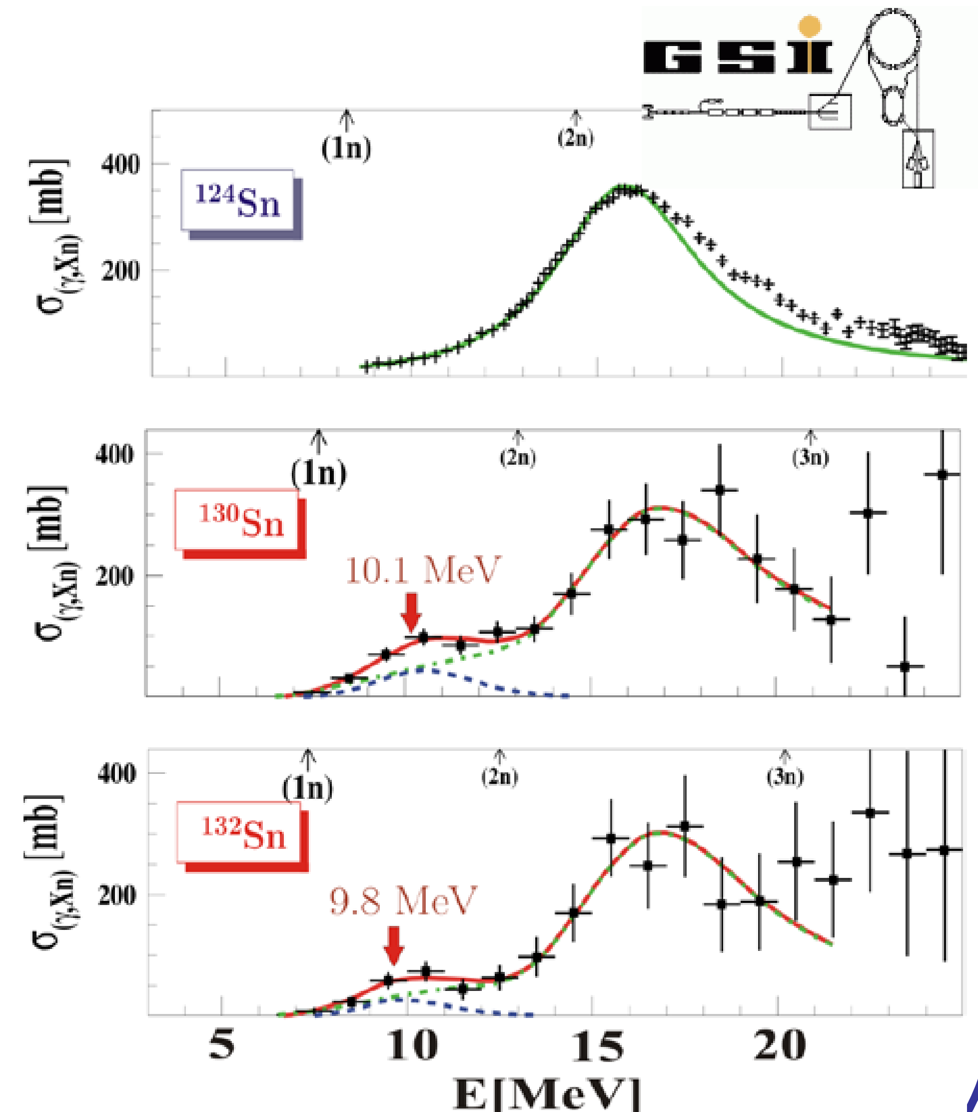
Krasznahorkay et al., Phys. Rev. Lett. 66, 1287 (1991).

There is a predictable correlation between the SDR cross section and the difference between the rms radii of the neutron and proton density distributions. By normalizing the results in the case of ^{120}Sn, data on neutron-skin thickness along the stable Sn isotopic chain were obtained, in good agreement with theoretical predictions.

A. Krasznahorkay et al., Phys. Rev. Lett. 82, 3216 (1999)

PYGMY DIPOLE RESONANCES AND NEUTRON SKIN





PDR corresponds to collective excitation mainly dominated by neutrons from the neutron skin. Therefore, the PDR should provide direct insight into the neutron skin properties.

P.Adrich et al., Phys. Rev. Lett. 95, 132501(2005)

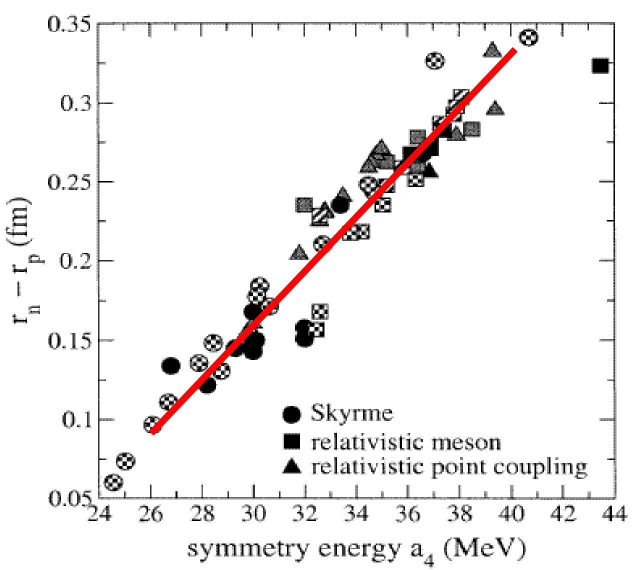
Symmetry energy $S_2(\rho)$ and neutron skin in ^{208}Pb

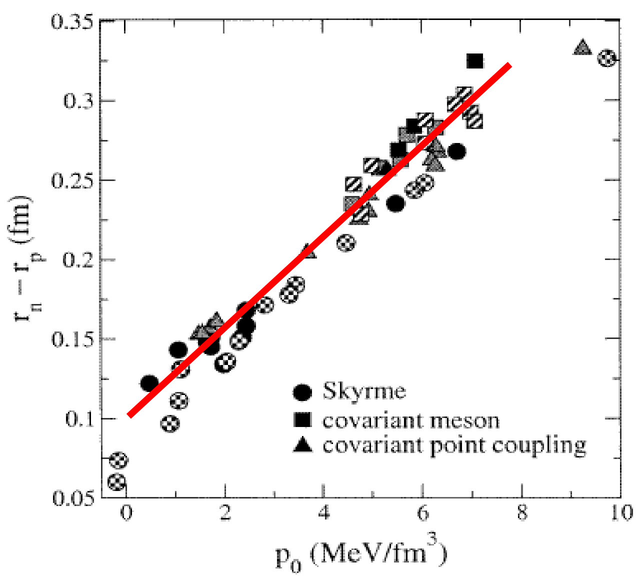
Energy per nucleon in asymmetric nuclear matter:

$$E(\rho,\alpha) = E(\rho,0) + S_2(\rho)\alpha^2 + O(\alpha^4)\,,\ \alpha = \frac{N-Z}{A}$$

$$S_2(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\alpha)}{\partial \alpha^2}\bigg|_{\alpha=0} =$$

$$= a_4 + \frac{p_0}{\rho_0^2}(\rho-\rho_0) + \frac{\Delta K_0}{18\rho_0^2}(\rho-\rho_0)^2 + \ldots$$





R.J.Furnstahl
NPA 706(2002)85-110

- strong linear correlation between neutron skin thickness and parameters a_4, p_0

- **Neutron skin could provide constraint on the nuclear symmetry energy**

NUCLEAR SYMMETRY ENERGY AND NEUTRON SKINS DERIVED FROM PYGMY DIPOLE RESONANCE

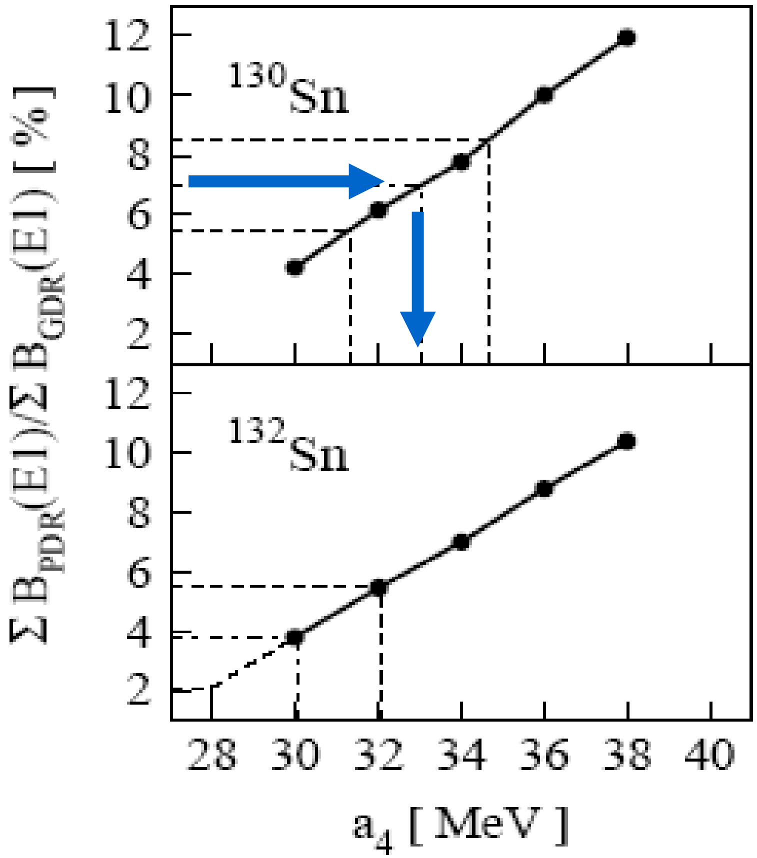
Precise knowledge of neutron-skin thickness could constrain the density dependence of S(ρ)

Pygmy-Strength (since related to skin) should do the same, but, experimentally, is accessed much easier !

Quantitative attempt to determine the neutron skin thickness by means of microscopic theory, using various density-dependent effective interactions and recent experimental data on PDR

RHB + RQRPA + GSI

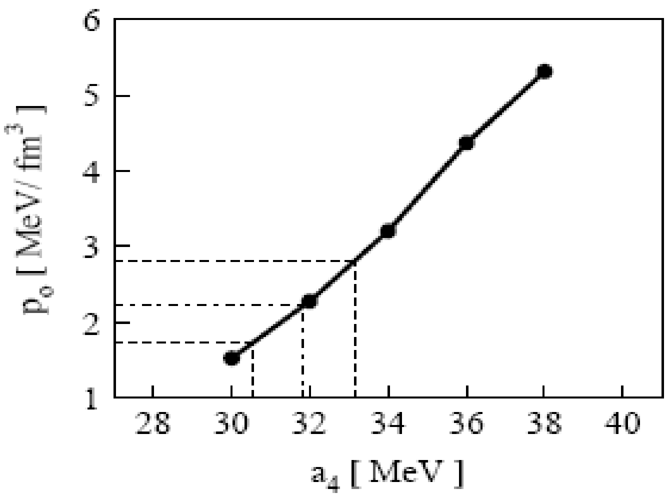
PDR strength versus a_4, p_o



RHB+RQRPA calculations and exp. data (GSI) for PDR strength

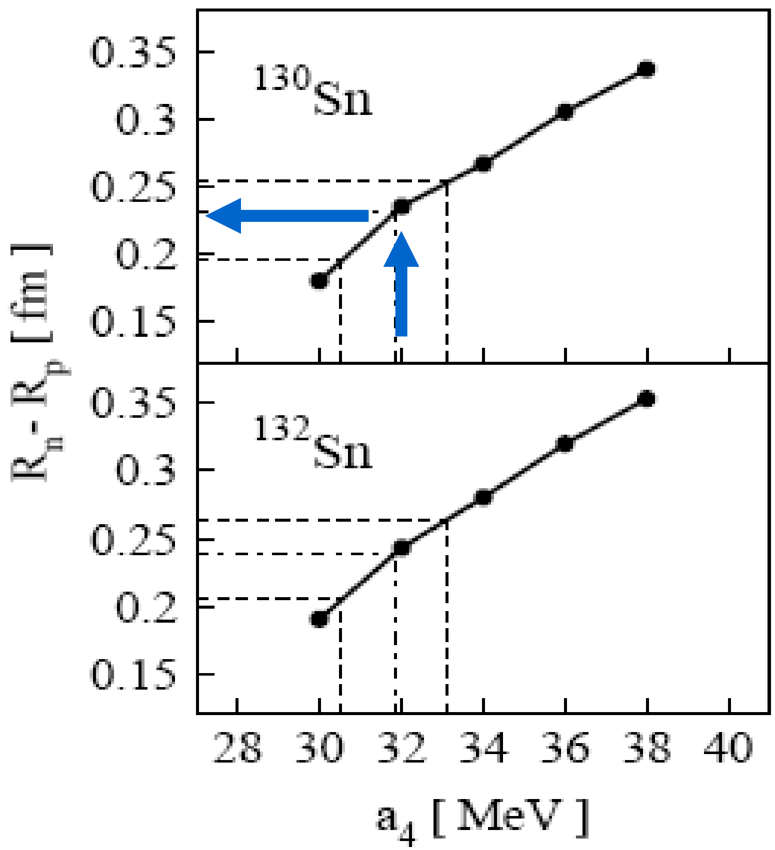
Result (*averaged* ^{130,132}*Sn) :*

a_4 = 32.0 ± 1.8 MeV



p_o = 2.3 ± 0.8 MeV/fm^3

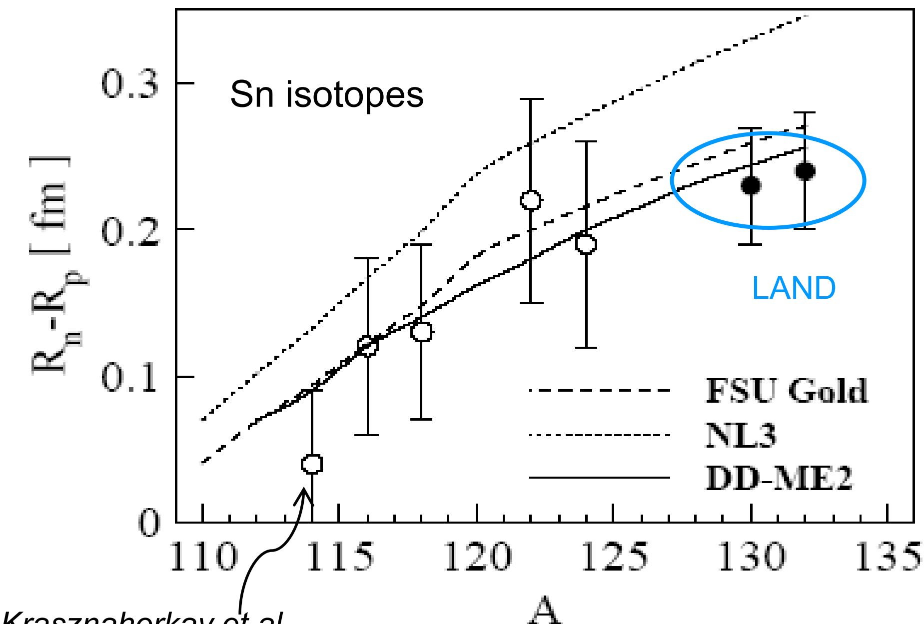
Neutron skin thickness



$R_n – R_p$:

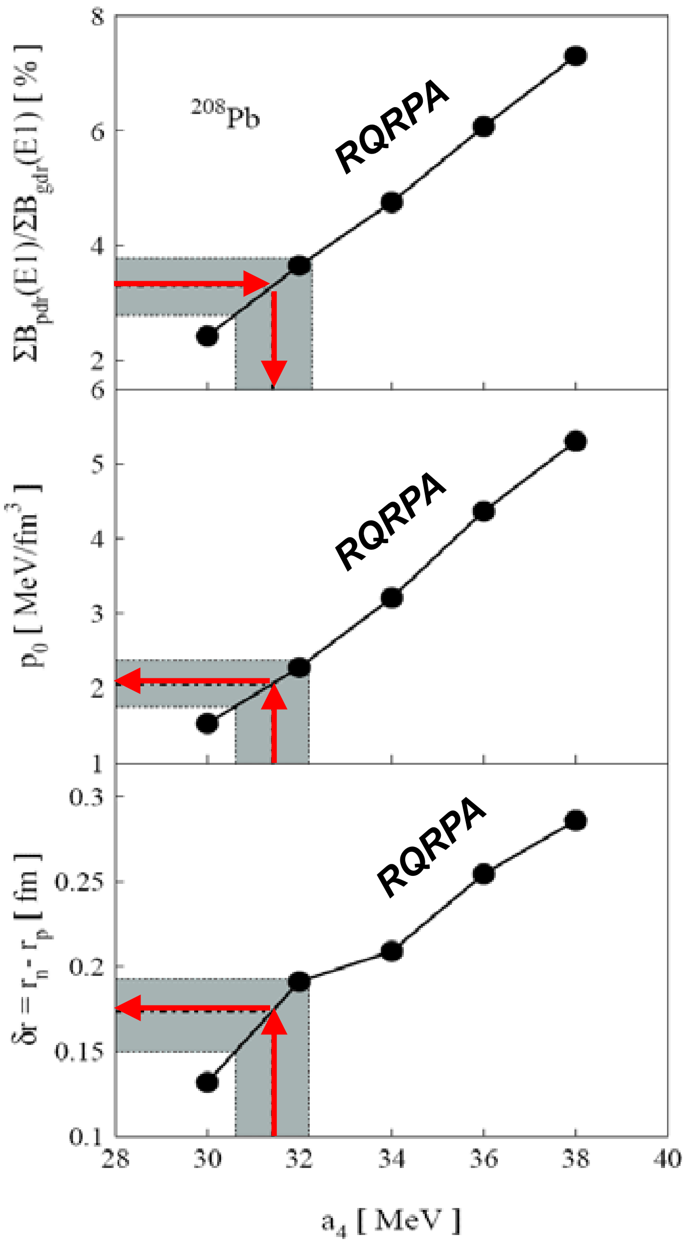
^{130}Sn: 0.23 ± 0.04 fm

^{132}Sn: 0.24 ± 0.04 fm



A.Krasznahorkay et al.
PRL 82(1999)3216

^{208}Pb analysis



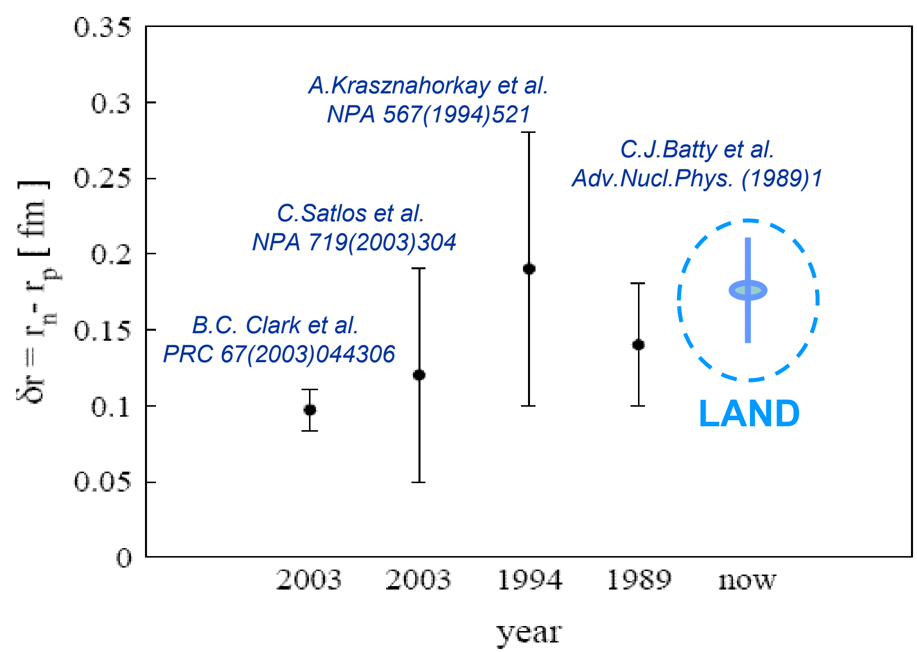
$\sum B_{pdr}(E1)=1.98\ e^2\ fm^2$

from N.Ryezayeva et al., PRL 89(2002)272501

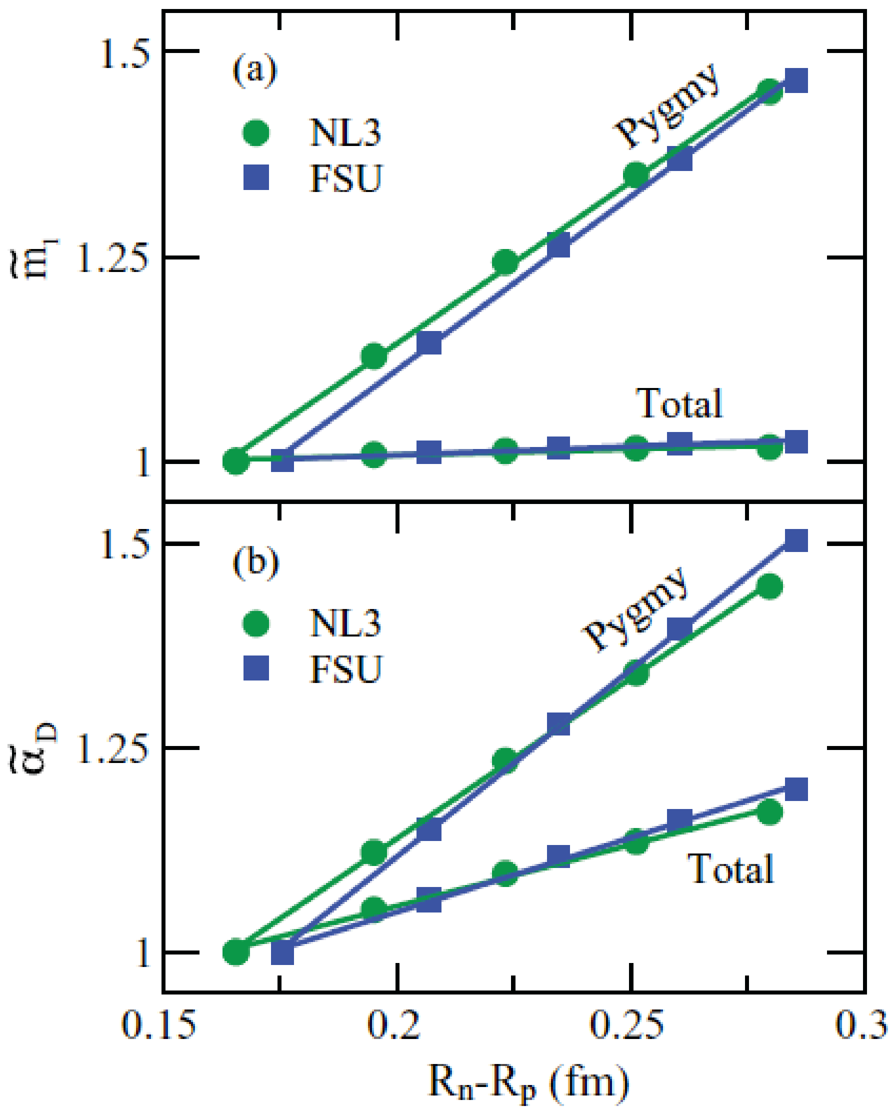
$\sum B_{gdr}(E1)=60.8\ e^2\ fm^2$

from A.Veyssiere et al.,NPA 159(1970)561

$$R_n - R_p = 0.18 \pm 0.035\ \ fm$$



Pygmy resonances and neutron skins

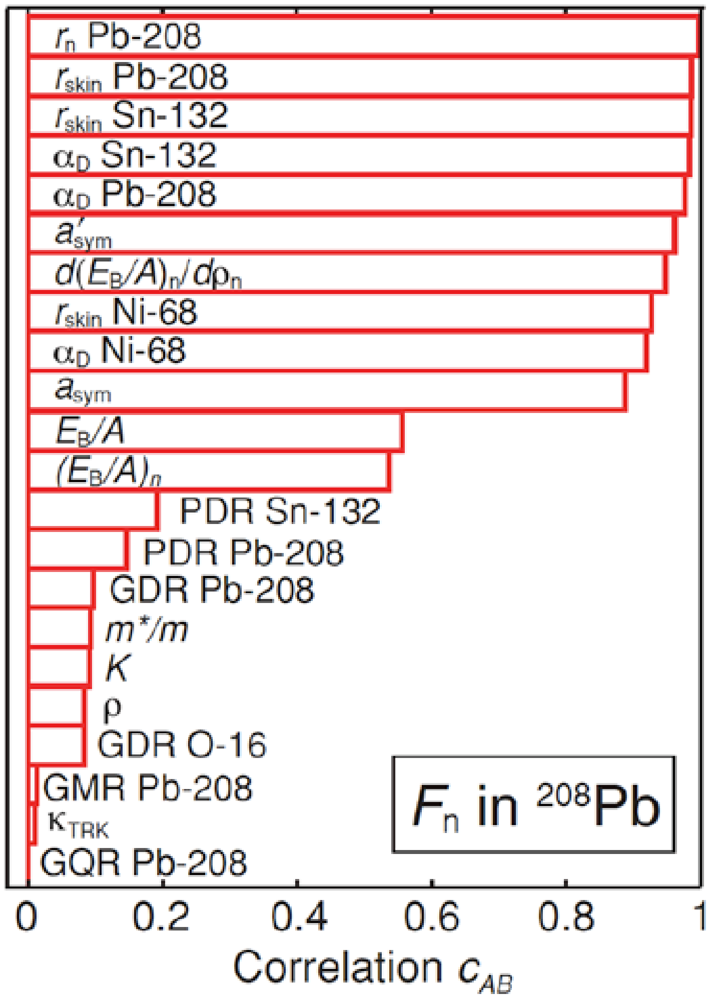


Energy weighted sum and dipole polarizability for ^{68}Ni as a function of the neutron-skin thickness of ^{208}Pb.

Families of NL3 and FSUGold parameterizations are obtained by changing the isovector parameters (for the isoscalar-isovector mixing term and the isovector term).

J. Piekarewicz, Phys. Rev. C 83, 034319 (2011)

Correlation of various observables with neutron form factor in ^{208}Pb, related to PREX experiment



SV-min (Skyrme type parameterization) embraces nuclear bulk properties (binding energies, surface thicknesses, charge radii, spin-orbit splittings, and pairing gaps) for selected semimagic nuclei.

lack of correlation between *Fn (or neutron skin)* and PDR strength; GMR, GDR, and GQR energies; and isoscalar and isovector effective mass, incompressibility, and saturation density

Strong correlation *Fn* with dipole polarizability:

$$\alpha_D = \frac{8\pi}{9} e^2 m_{-1}$$

P.-G. Reinhard, W. Nazarewicz, Phys. Rev. C 81, 051303 (2010)

Covariance analysis and correlations between observables

Microscopic models based on NEDF include number of parameters

$$\mathbf{p} = (p_1, p_2, p_3, ..., p_N)$$

The optimal set of parameters $\mathbf{p_0}$ is obtained using a set of observables in a least square fit with the quality measure:

$$\chi^2(\mathbf{p}) = \sum_{\mathcal{O}} \left(\frac{\mathcal{O}^{(\mathrm{th})}(\mathbf{p}) - \mathcal{O}^{(\mathrm{exp})}}{\Delta\mathcal{O}} \right)^2$$

Near the minimum, the *χ2* landscape is given by a confidence ellipsoid

$$\chi^2(\mathbf{p}) - \chi_0^2 \approx \sum_{i,j=1}^{N} (p_i - p_{i,0}) \mathcal{M}_{ij} (p_j - p_{j,0})$$

$$\mathcal{M}_{ij} = \tfrac{1}{2} \partial_{p_i} \partial_{p_j} \chi^2 |_{\mathbf{p}_0}$$

The physically reasonable domain **p** is defined as that multitude of parameters around $\mathbf{p_0}$ that fall inside the covariance ellipsoid $\chi^2 = \chi_0^2 + 1$

Covariance analysis and correlations between observables

Consider two observables A, B calculated within the model, they are also functions of the model parameters

$$A = A(\mathbf{p}) \qquad\qquad B = B(\mathbf{p})$$

Assuming they are smoothly varying with **p**, the covariance between the observables A and B:

$$\overline{\Delta A\, \Delta B} = \sum_{ij} \partial_{p_i} A (\hat{\mathcal{M}}^{-1})_{ij} \partial_{p_j} B$$

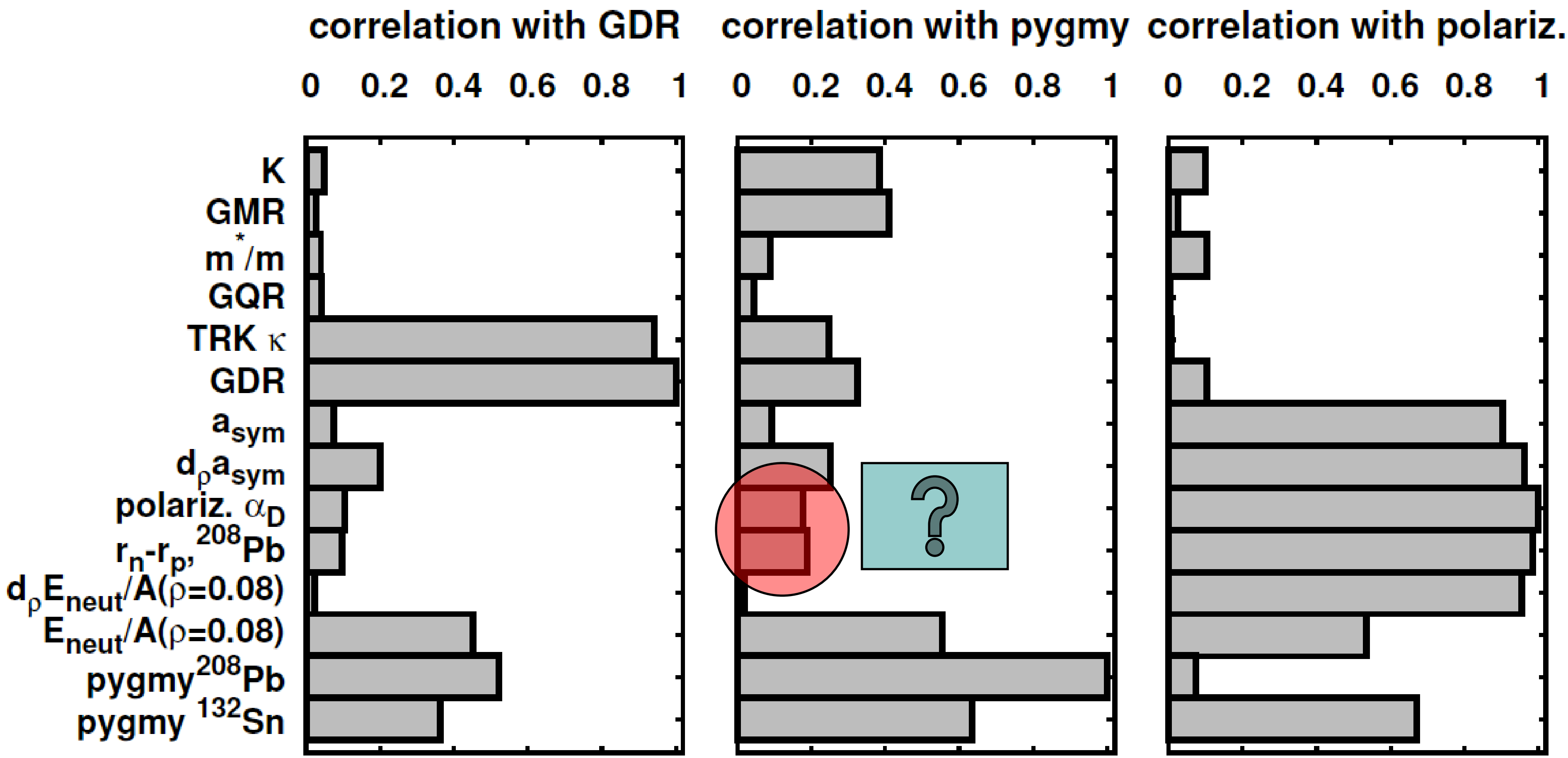
Variance $\overline{\Delta^2 A}$ and $\overline{\Delta^2 B}$ define uncertainties of each observable.

Pearson product-moment correlation coefficient

$$c_{AB} = \frac{|\overline{\Delta A\, \Delta B}|}{\sqrt{\overline{\Delta A^2}\ \overline{\Delta B^2}}}$$

provides a measure of the correlation (linear dependence) between two variables A and B, giving a value between +1 and −1 inclusive.

Covariance analysis and correlations between observables



P.-G. Reinhard, W. Nazarewicz, “Co-Variances in connection to χ^2 fitting” (2011)

Covariance analysis based on RNEDF

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_m + \mathcal{L}_{int}$$

- the Lagrangian of the free nucleon: $\mathcal{L}_N = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi$

- the Lagrangian of the free meson fields and the electromagnetic field:

$$\begin{aligned}\mathcal{L}_m &= \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ &\quad -\frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

- minimal set of interaction terms:

$$\mathcal{L}_{int} = -\bar{\psi}\Gamma_\sigma\sigma\psi - \bar{\psi}\Gamma_\omega^\mu\omega_\mu\psi - \bar{\psi}\vec{\Gamma}_\rho^\mu\vec{\rho}_\mu\psi - \bar{\psi}\Gamma_e^\mu A_\mu\psi.$$

with the vertices:

$$\Gamma_\sigma = g_\sigma, \quad \Gamma_\omega^\mu = g_\omega\gamma^\mu, \quad \vec{\Gamma}_\rho^\mu = g_\rho\vec{\tau}\gamma^\mu, \quad \Gamma_e^m = e\frac{1-\tau_3}{2}\gamma^\mu$$

Density dependence of the model

A) an effective density dependence introduced through a non-linear potential:

$$U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{g_2}{3}\sigma^3 + \frac{g_3}{4}\sigma^4$$

model parameters: meson masses m_σ, m_ω, m_ρ, meson-nucleon coupling constants g_σ, g_ω, g_ρ, nonlinear self-interactions coupling constants g_2, g_3

PARAMETERIZATION: NL3

B) the medium dependent meson-nucleon couplings g_σ, g_ω, g_ρ

-> functions of vector density: $\rho_v = \sqrt{j_\mu j^\mu}$ $\quad j_\mu = \bar{\psi}\gamma_\mu\psi$

$$g_i(\rho) = g_i(\rho_{\text{sat}}) f_i(x)$$

$$f_i(x) = a_i \frac{1 + b_i (x + d_i)^2}{1 + c_i (x + d_i)^2}$$

$$i = \sigma, \omega$$

$$g_\rho(\rho) = g_\rho(\rho_{\text{sat}})\, e^{-a_\rho (x-1)}$$

$$x = \rho / \rho_{\text{sat}}$$

model parameters: meson masses + parameters of vertex functions

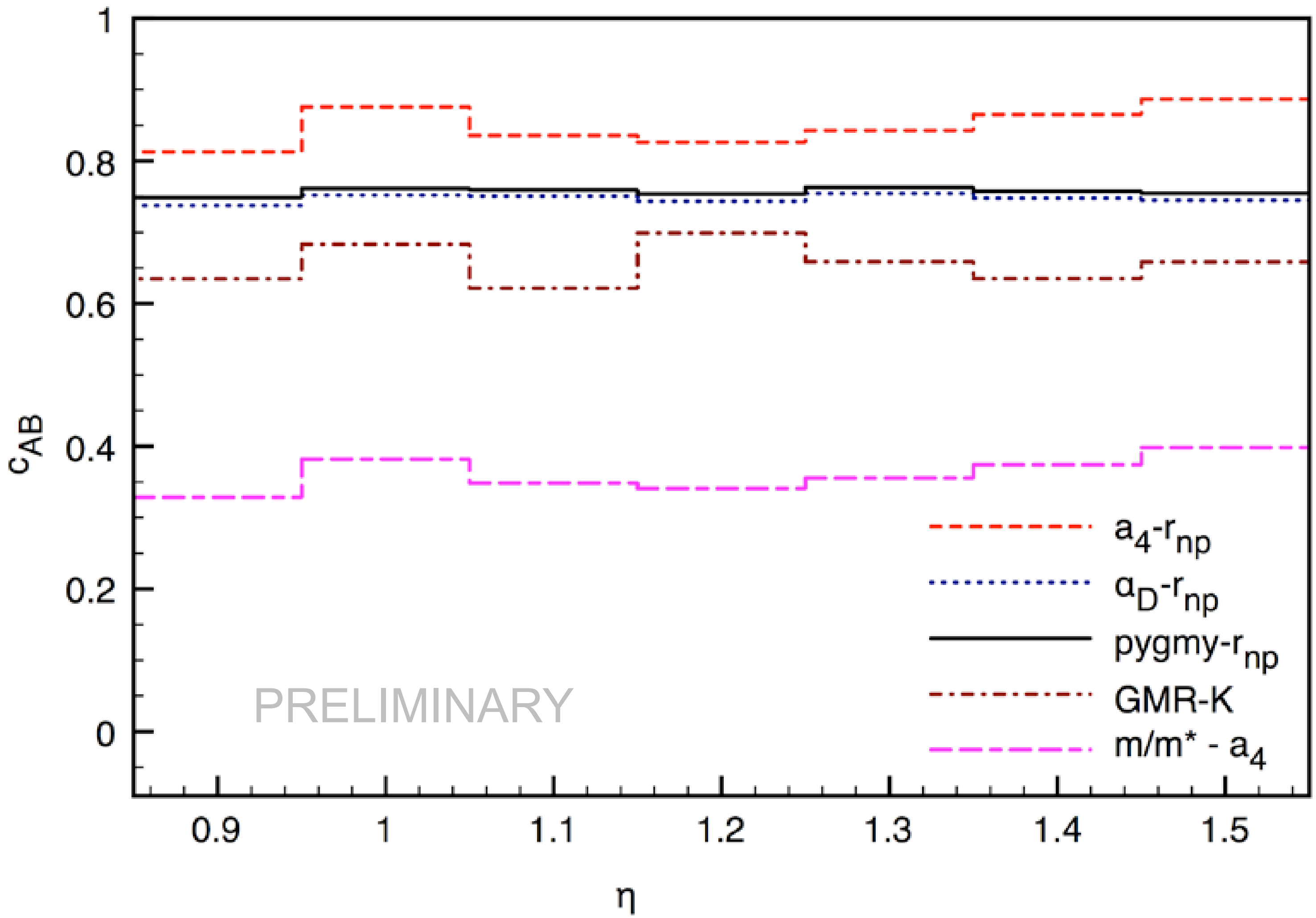
PARAMETERIZATION: DD-ME2

Correlations c_{AB} between various quantities

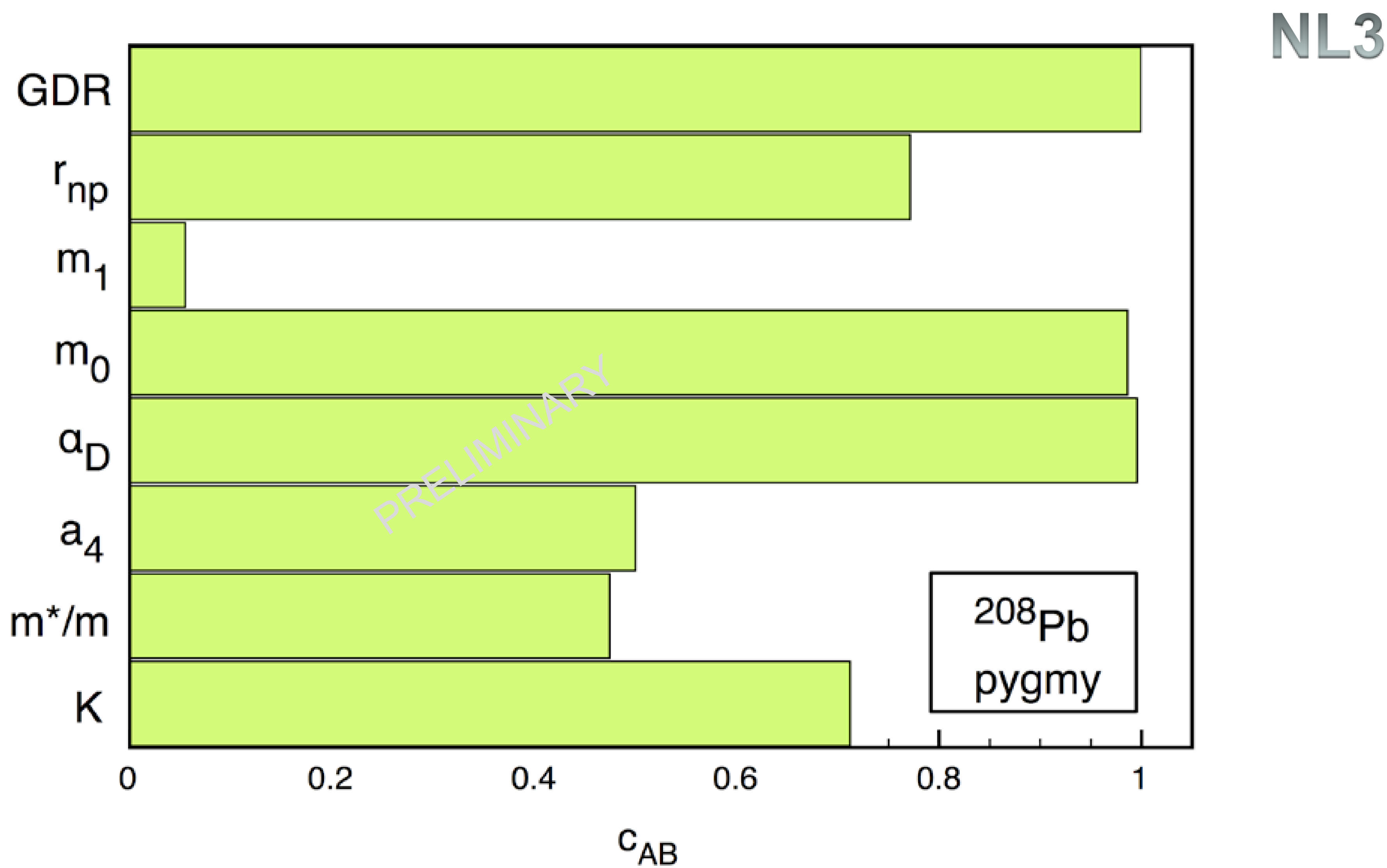
$$p_i = p_{0,i} + \eta \delta p_i$$

$$\chi^2(\mathbf{p}) - \chi_0^2 = 1 \longrightarrow \eta = 1$$

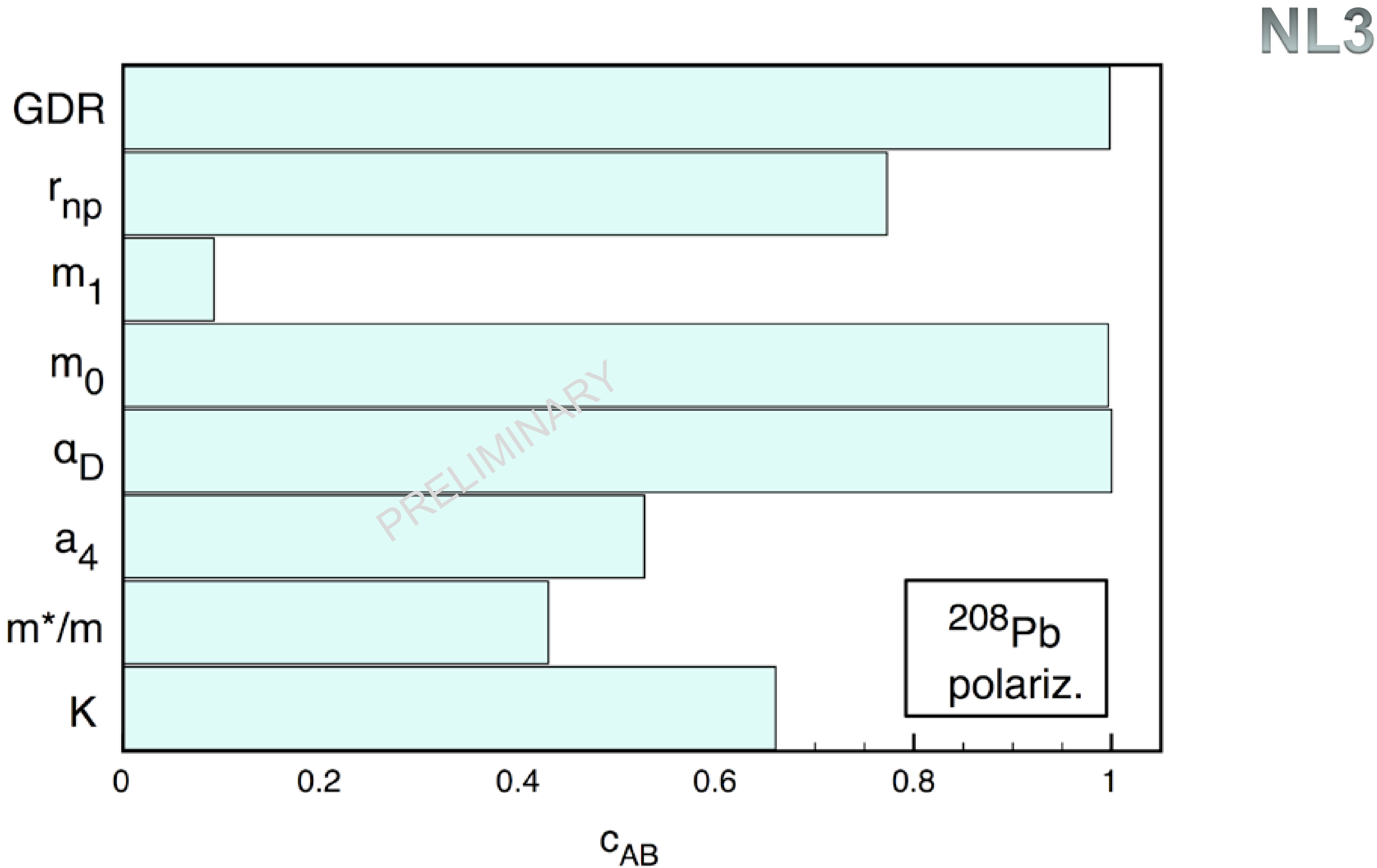
NL3



Correlations c_{AB} between pygmy strength and various quantities

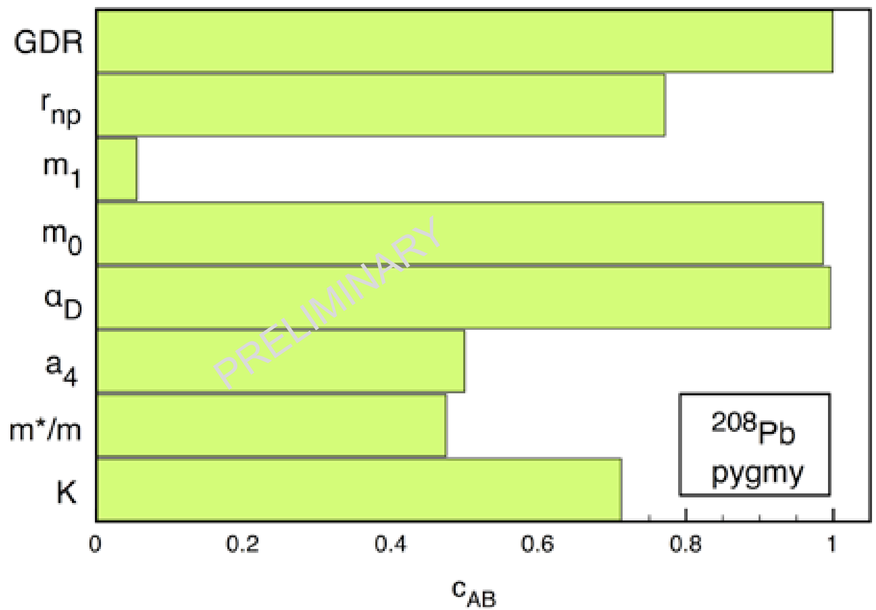


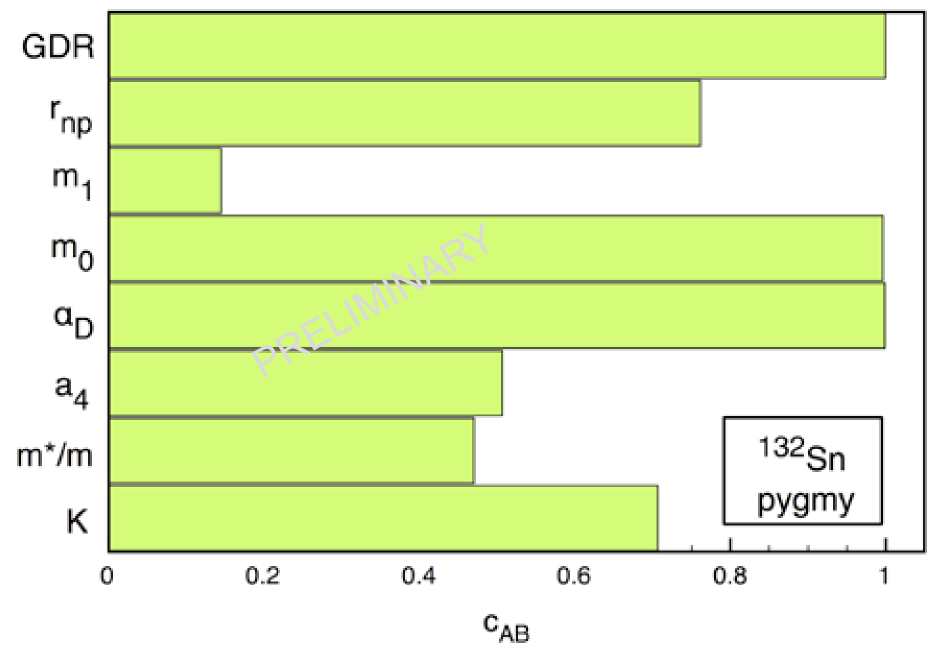
Correlations c_{AB} between dipole polarizability and various quantities



Correlations c_{AB} for the pygmy strength in ^{132}Sn and ^{208}Pb

NL3





CONCLUSIONS

What is physical content of new observables, how to connect with the properties which are difficult to assess experimentally?

Statistical analysis based on covariances and Skyrme SV-min interaction indicates that dipole polarizability represents an observable correlated to the neutron skin, while correlations between pygmy strength and neutron skin are absent (P.-G. Reinhard, W. Nazarewicz).

Preliminary NL3-based covariance analysis indicates the same correlations between the pygmy strength and dipole polarizability with respect to the size of the neutron skin. Work on other effective interactions in progress.

How reliable are the covariance analysis and correlations? Model dependence?

Physical arguments vs. statistical analysis?