What is true:
Melting of nuclei or transfer of nucleons in the production of superheavy nuclei?

That is an open question.

In this talk I want to give certain explanations of our understanding of the fusion dynamics.

Presently one needs more experimental data to have an unique decisive answer to this question.

Most of this work has be done in collaboration with

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Joint Institute for Nuclear Reactions in Dubna

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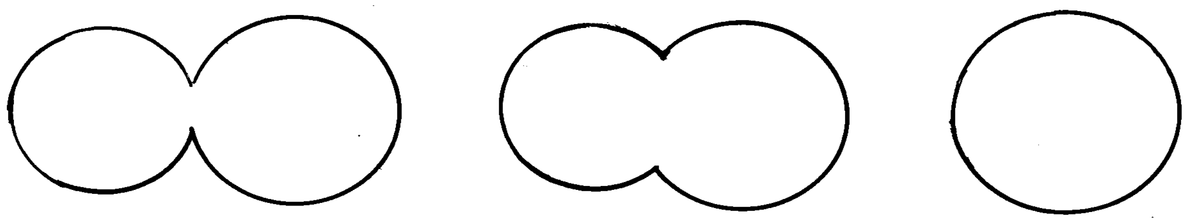
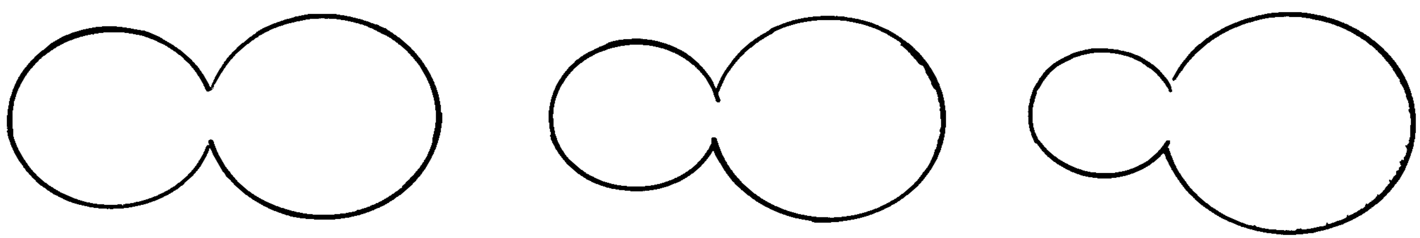
1. Introduction

The fussion of two nuclei to a superheavy nucleus can be thought as melting process



or a nucleon transfer process

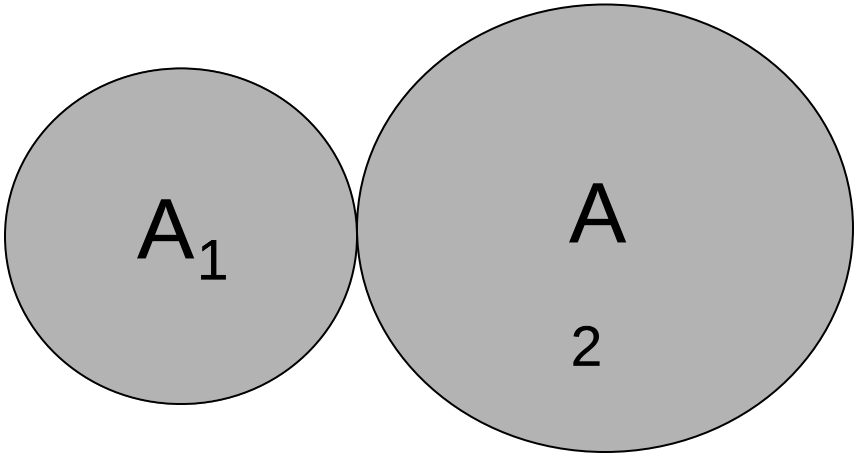


This talk will discuss the two possibilities.

Two important degrees of freedom:

1. Relative motion, described by R

2. Mass asymmetry motion, described by $h = (A_1 - A_2) / (A_1 + A_2)$



$\eta = 0$ for $A_1 = A_2$, $\eta = \pm 1$ for A_1 or $A_2 = 0$

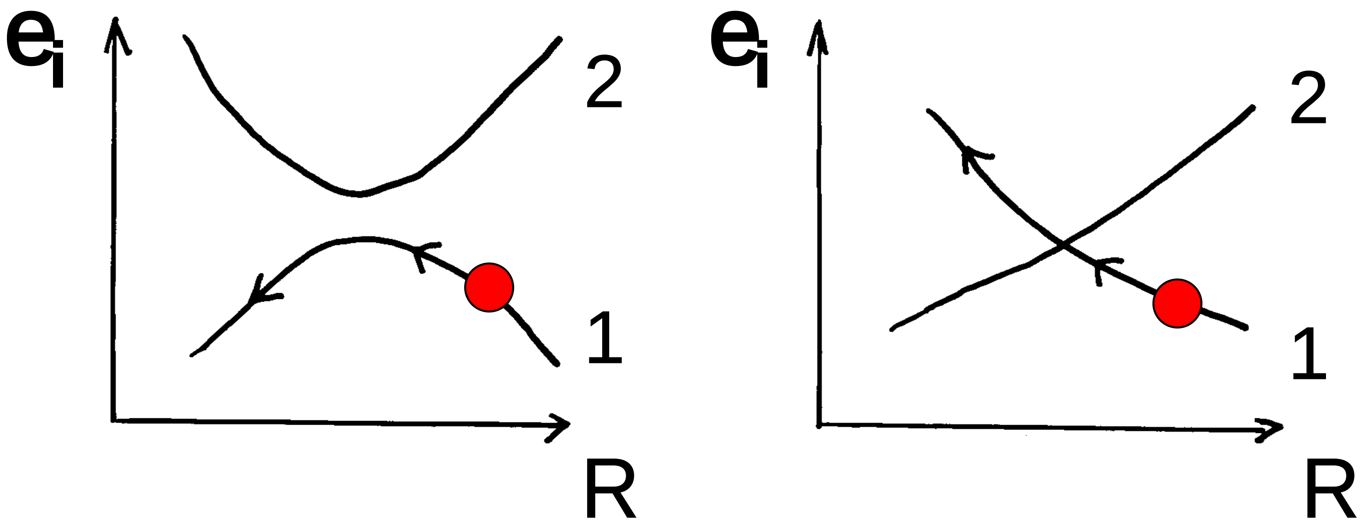
2. Models with adiabatic and diabatic potentials for the relative motion

Calculation of internuclear potential semiclassically with Strutinsky formalism

$$U = U_{\text{liquid drop}} + d U_{\text{shell}} \cdot$$

The potential includes shell effects.
$d U_{\text{shell}}$ can be calculated with an adiabatic or a diabatic two-center shell model.

Explanation with two-center shell model:



adiabatic model diabatic model

Velocity between nuclei leads to diabatic occupation of single-particle levels; behind is the Pauli principle between the nuclei.

Here we use the two-center shell model of Maruhn and Greiner (1973)

Parameters are:

length

$l = L/(2R_0)$,

mass asymmetry h,

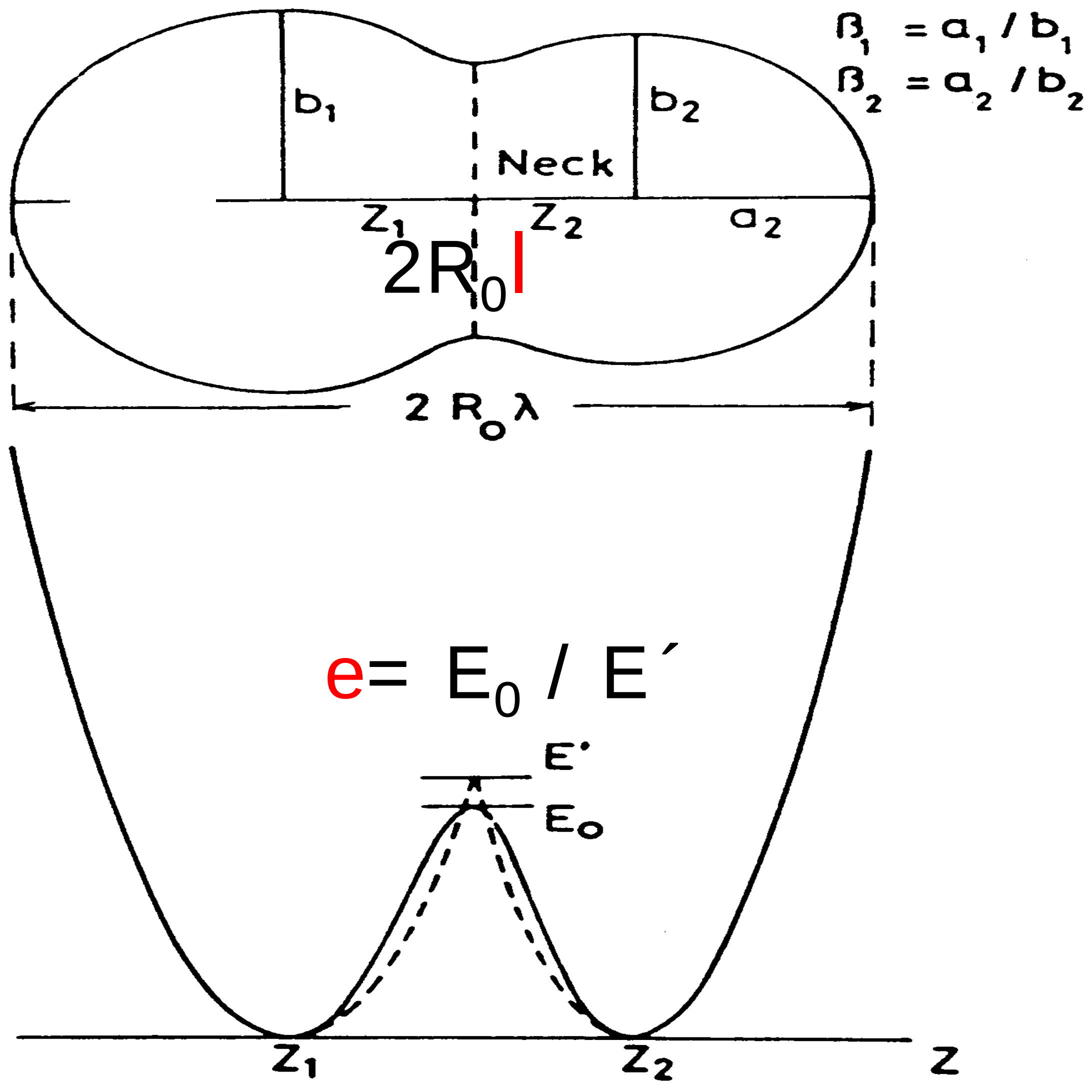
deformations

$b_i = (a/b)_i$

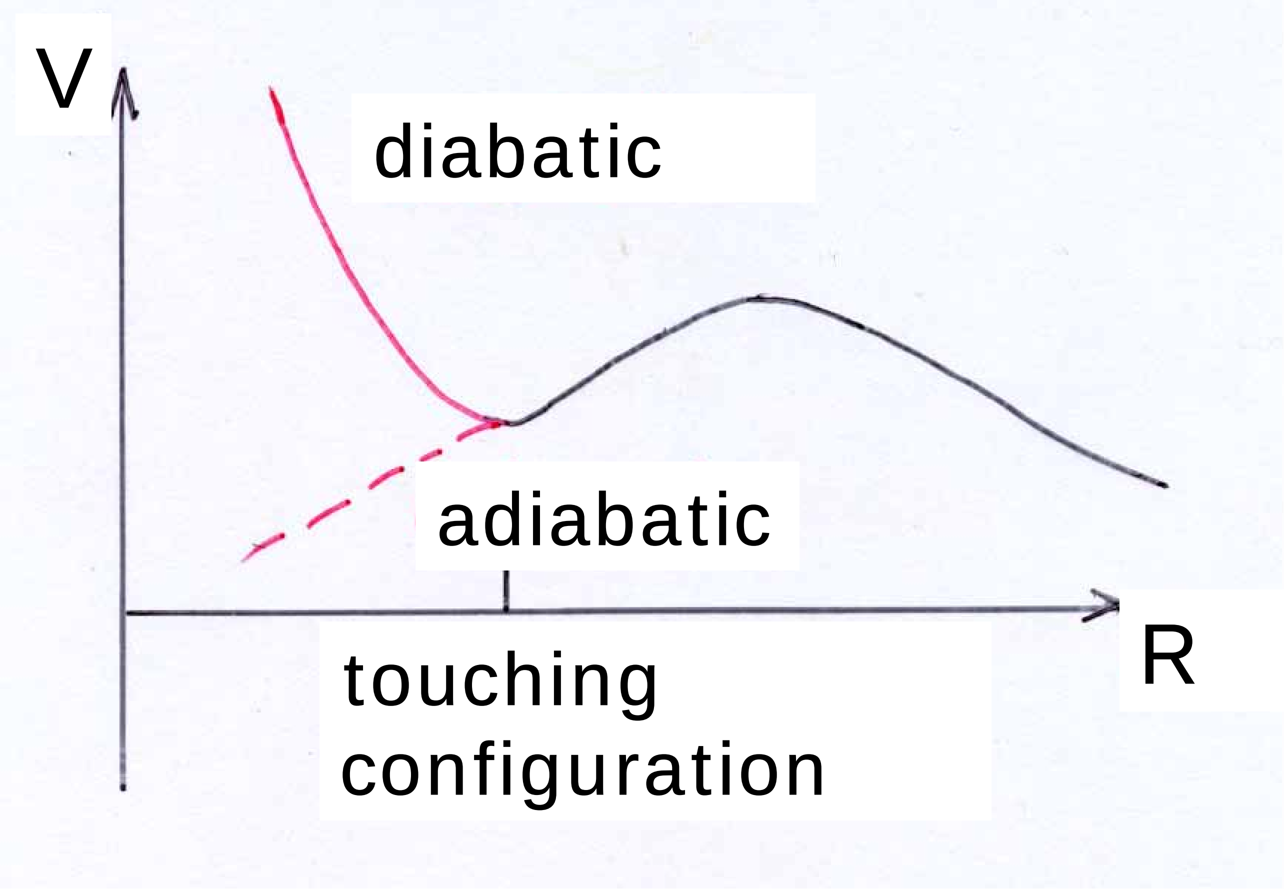
(ratio of semiaxes),

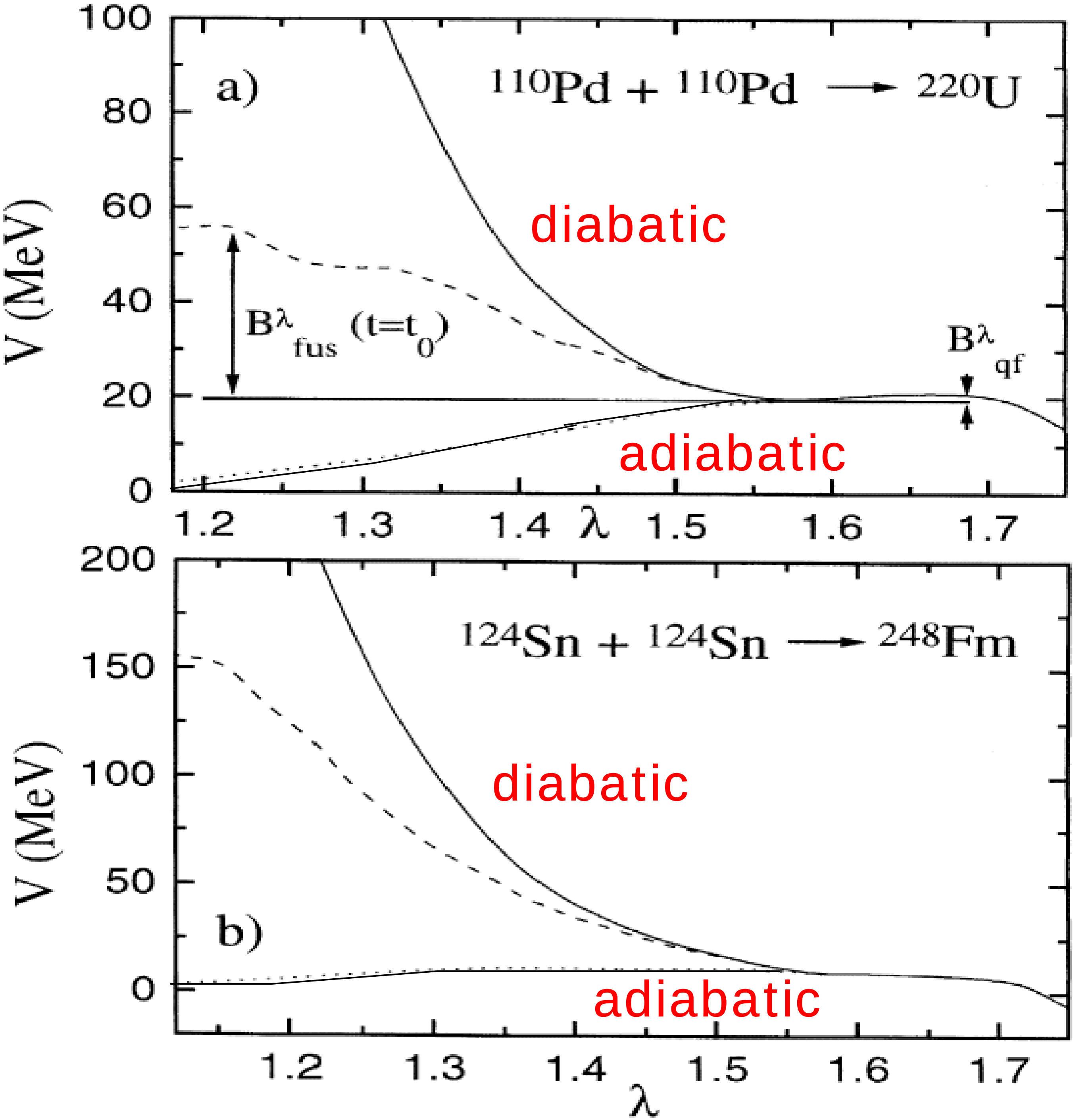
neck parameter

$e = E_0 / E'$ (ratio of barriers).



Description of fusion dynamics depends strongly whether adiabatic or diabatic potential energy surfaces are assumed.





a)
$^{110}Pd + ^{110}Pd \rightarrow ^{220}U$
diabatic
adiabatic
B^{λ}_{fus} $(t=t_0)$
B^{λ}_{qf}
V (MeV)
λ
b)
$^{124}Sn + ^{124}Sn \rightarrow ^{248}Fm$
diabatic
adiabatic
V (MeV)
λ

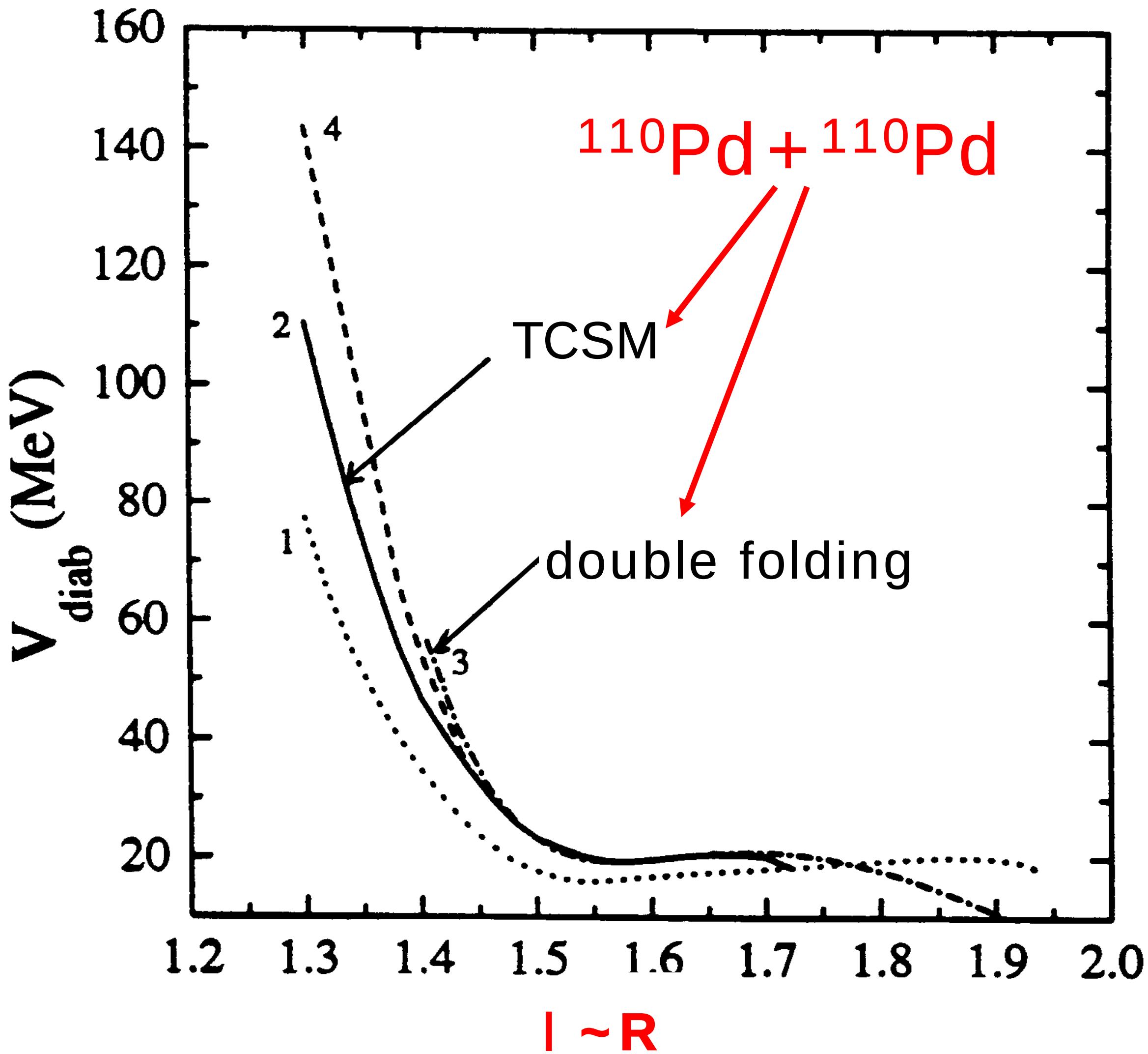


Fig.2. Diabatic potentials for the systems:

(1) $^{100}Mo + ^{100}Mo$ (TCSM)

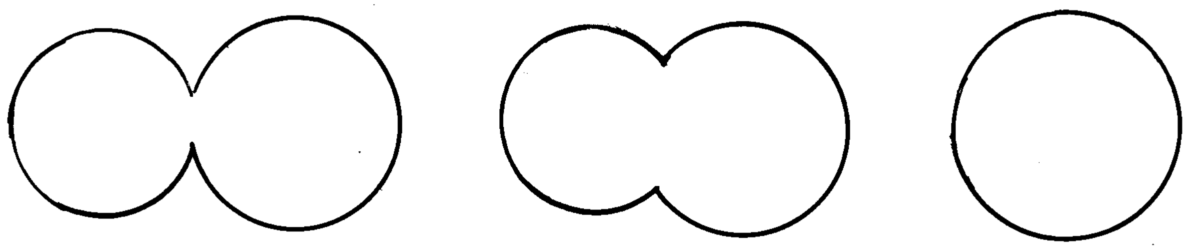
(2) $^{110}Pd + ^{110}Pd$ (TCSM)

(3) $^{110}Pd + ^{110}Pd$ (double folding)

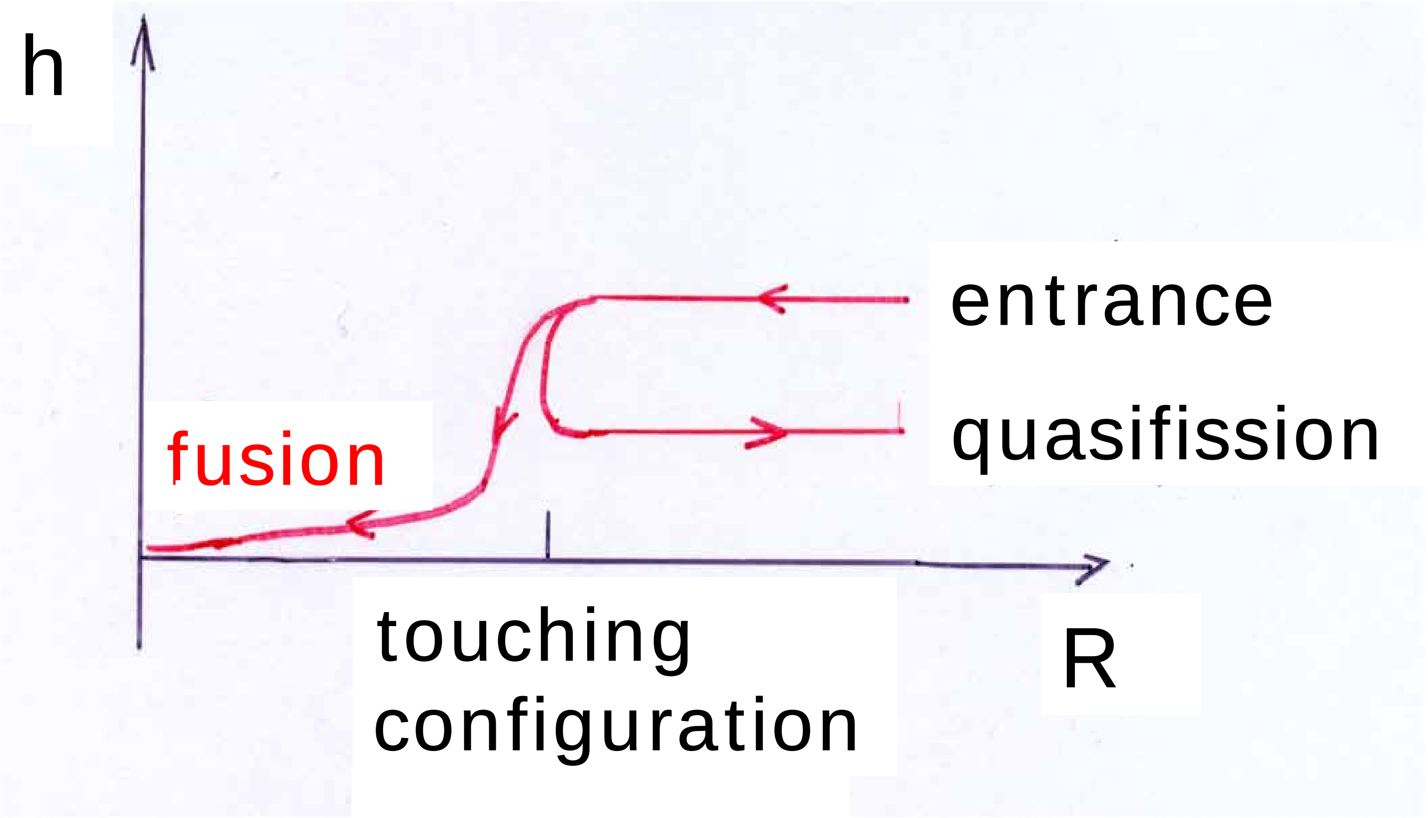
(4) Neck parameter ε is diminished with decreasing λ

a) Models using adiabatic potentials

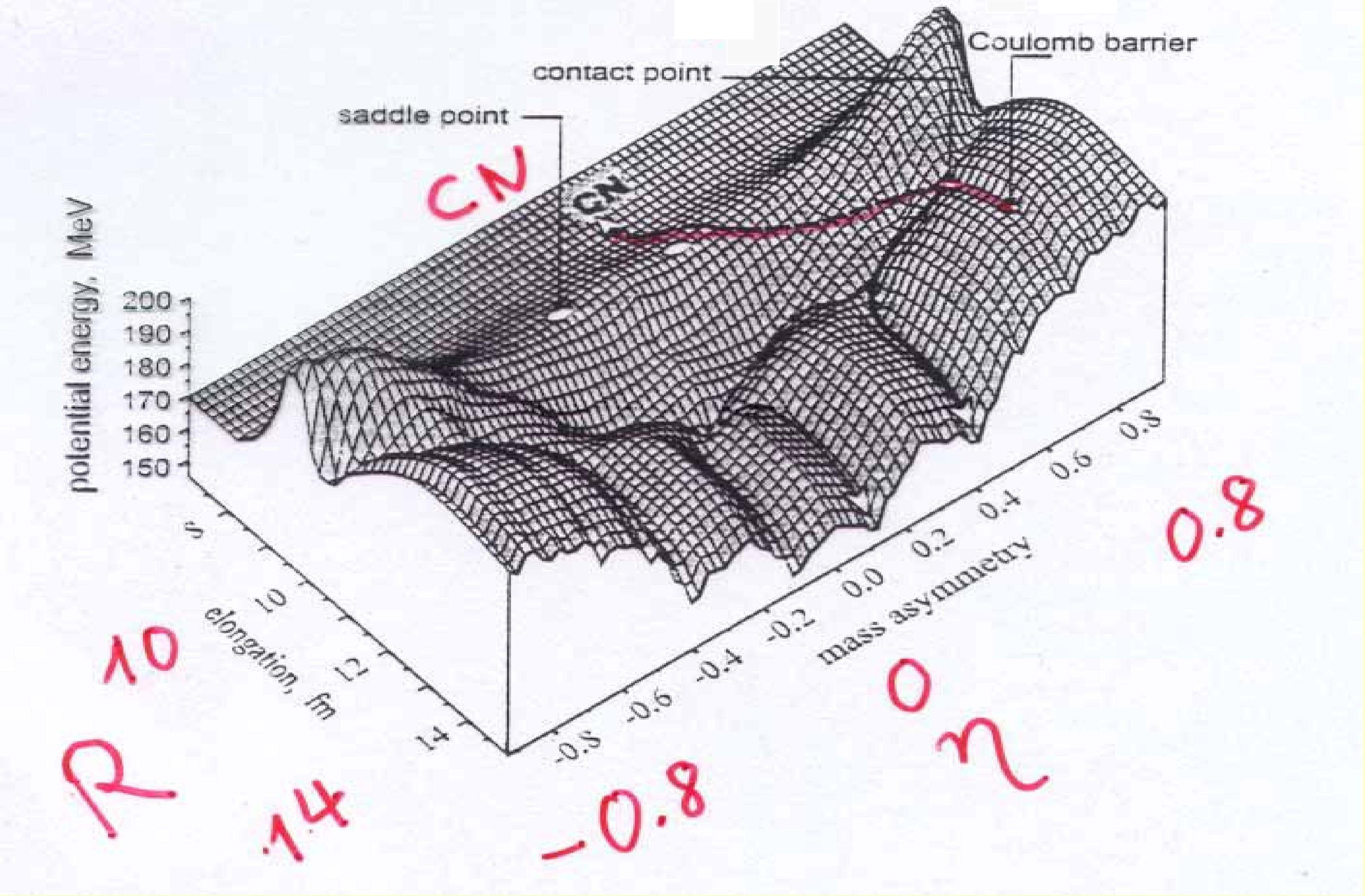
Minimization of potential energy, essentially adiabatic dynamics in the internuclear distance, nuclei melt together.



Large probabilities of fusion for producing nuclei with similar projectile and target nuclei (h=0).

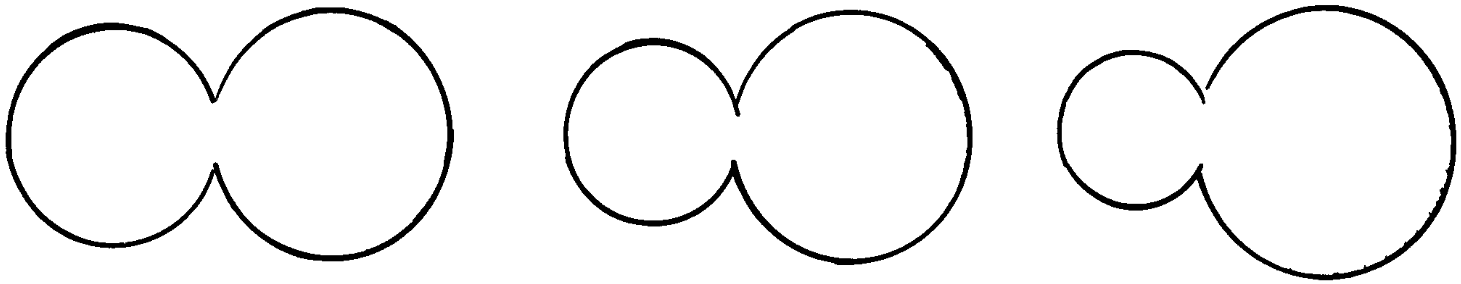

h
entrance
quasifission
fusion
touching
configuration
R

$^{48}Ca + ^{246}Cm$ (from Zagrebaev)

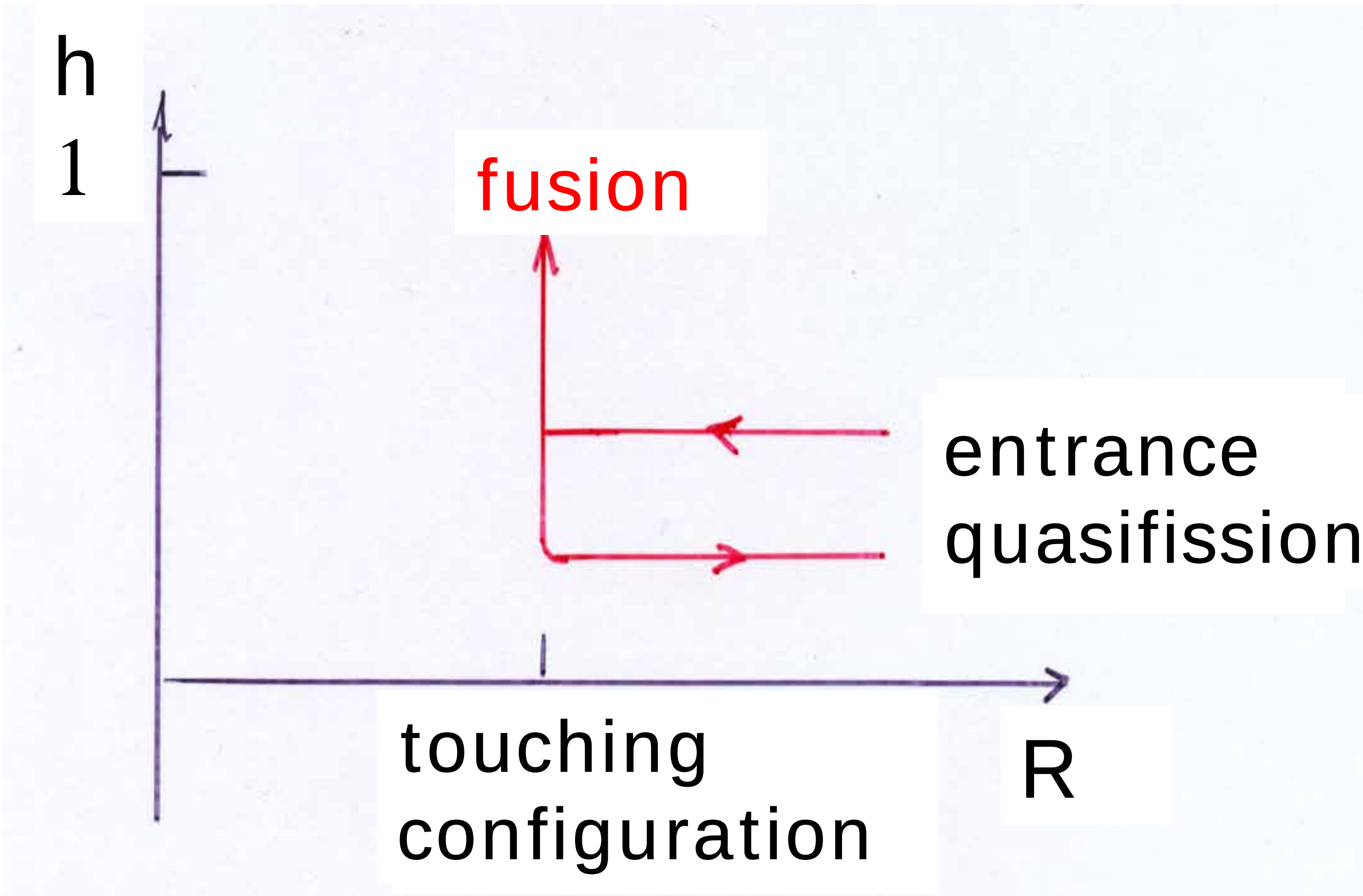


b) Dinuclear system (DNS) concept

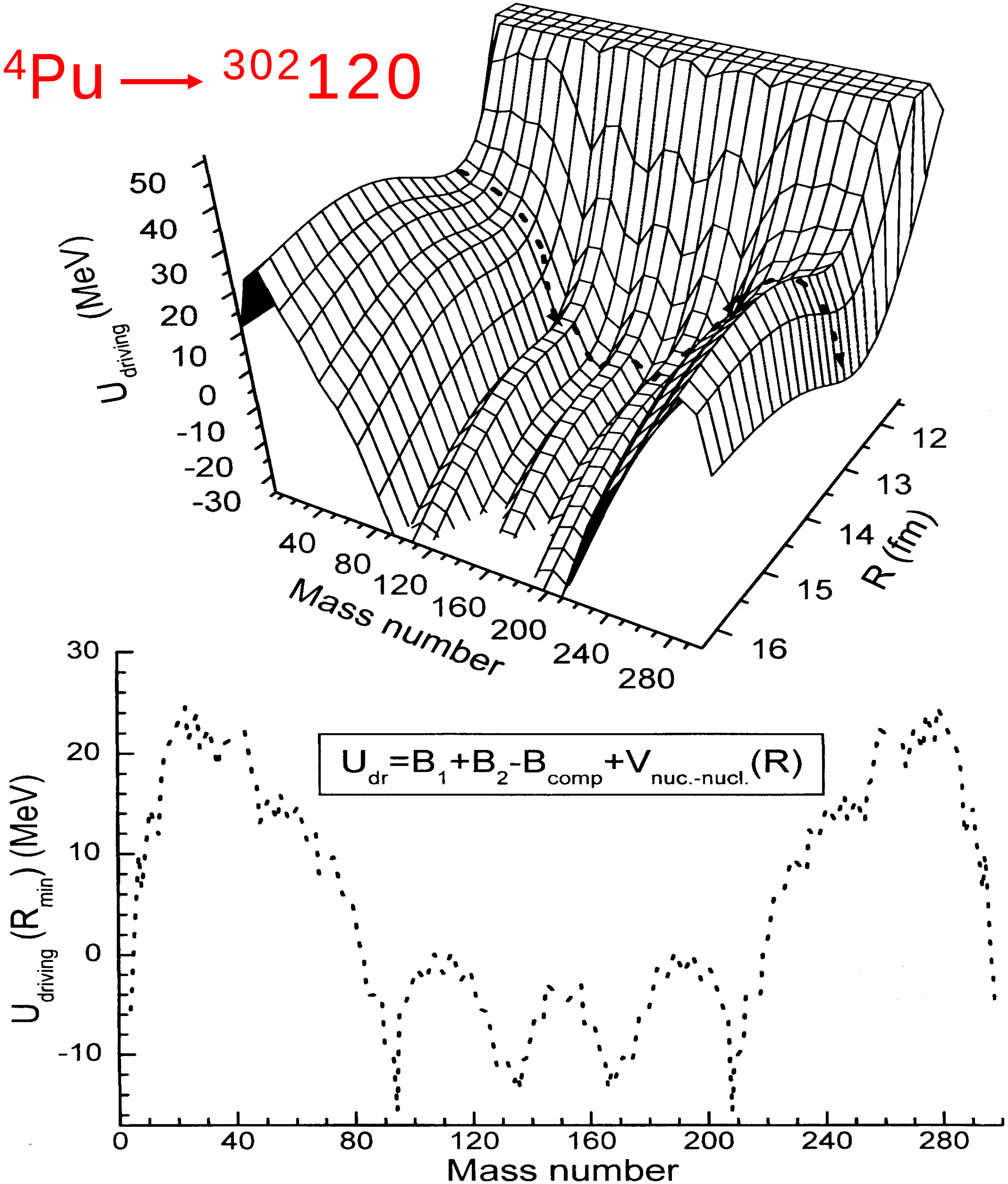
Fusion by transfer of nucleons between the nuclei (idea of V. Volkov, also von Oertzen), mainly dynamics in mass asymmetry degree of freedom, use of diabatic potentials, e.g. calculated with the diabatic two-center shell model.



$h \to 1$


h
1
fusion
entrance
quasifission
touching
configuration
R

$^{58}Fe + ^{244}Pu \longrightarrow ^{302}120$

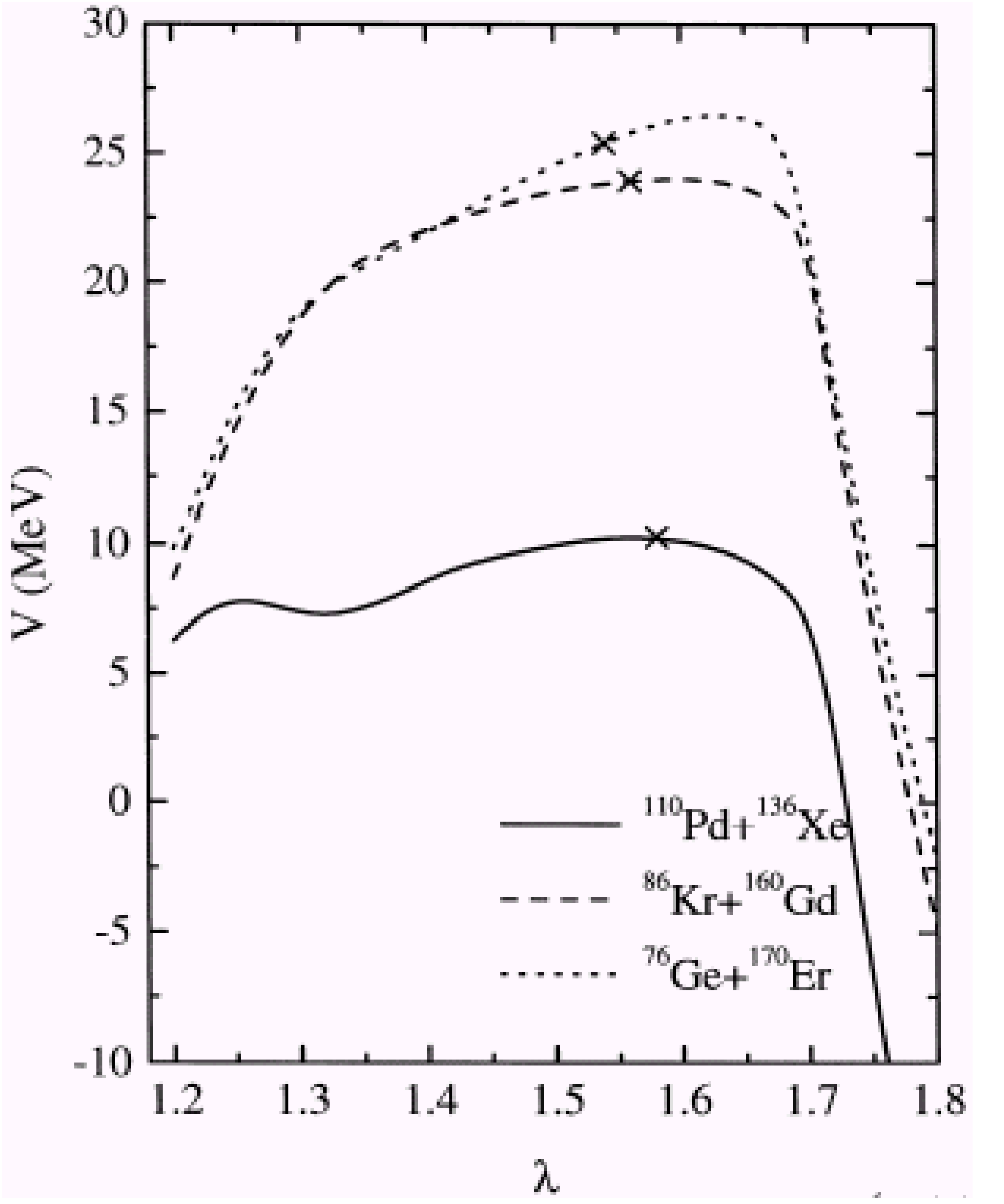


3. Comparison of fusion probabilities calculated with adiabatic and diabatic models

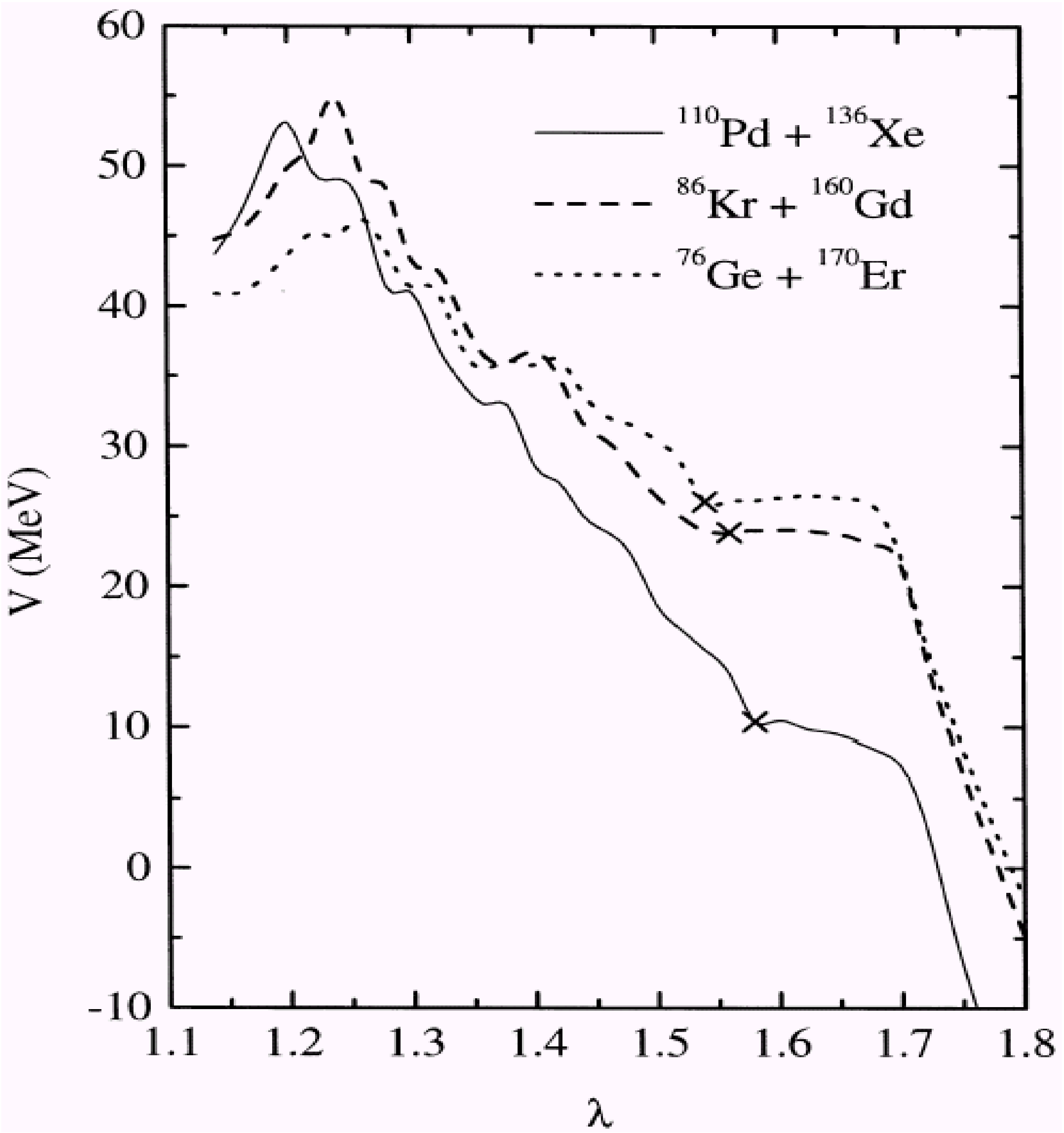
Principle statement: If adiabatic potentials are used with more and more additional degrees of freedom, the kinetic energy of relative motion is transfered into excitation energy and the system sticks together in the minimum of the internuclear potential. Then one has nucleon transfer as in the DNS model up to the formation of the compound nucleus.

Here: Examples of a simple adiabatic and diabatic description leading to ^{246}Fm

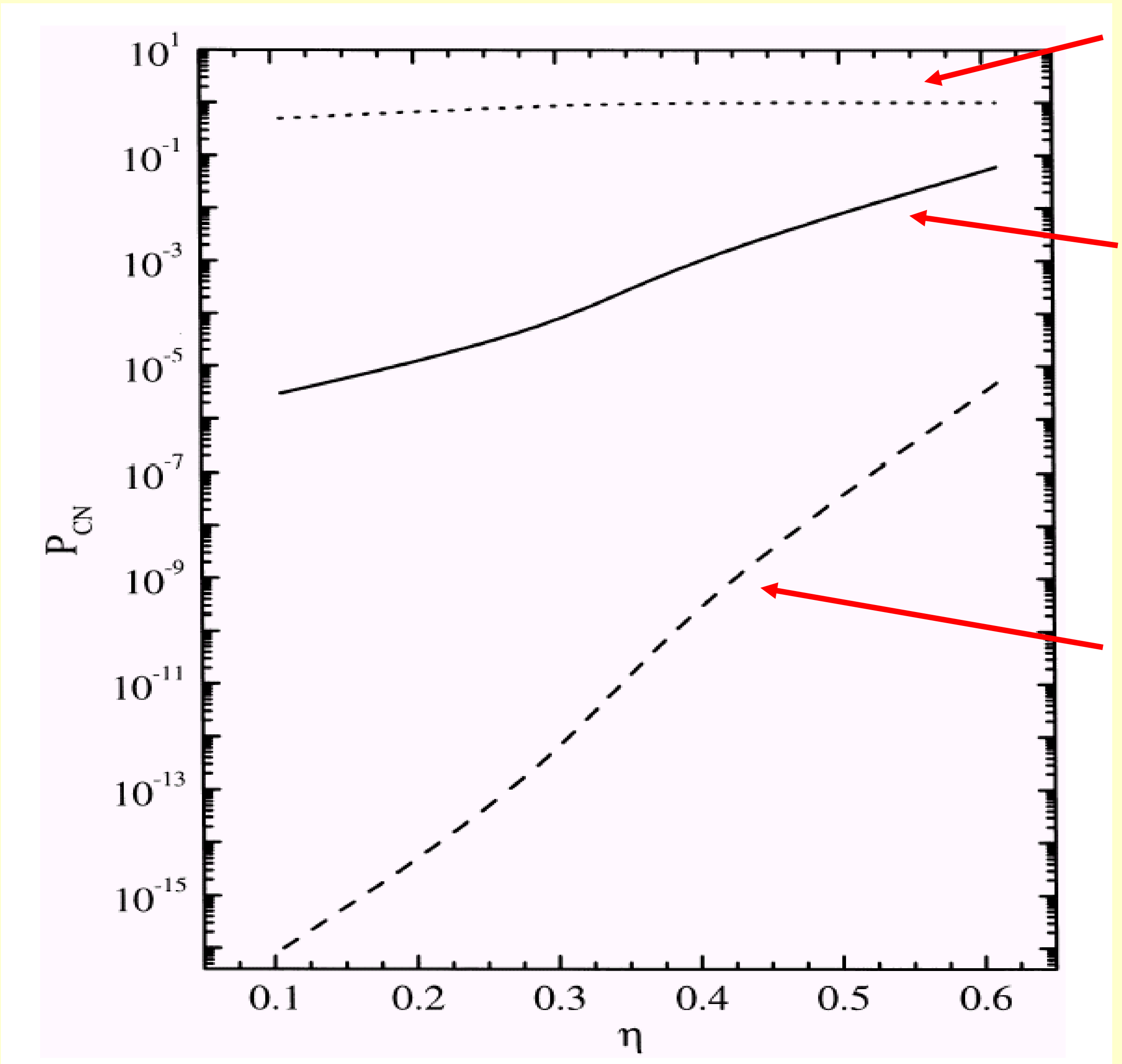
Adiabatic potentials for different combinations leading to ^{246}Fm; e=0.75



Dynamical diabatic potentials; e=0.75



Fusion probability P_{CN} leading to ^{246}Fm (E*=30MeV)



adiabatic in l

fusion in h » experim. data

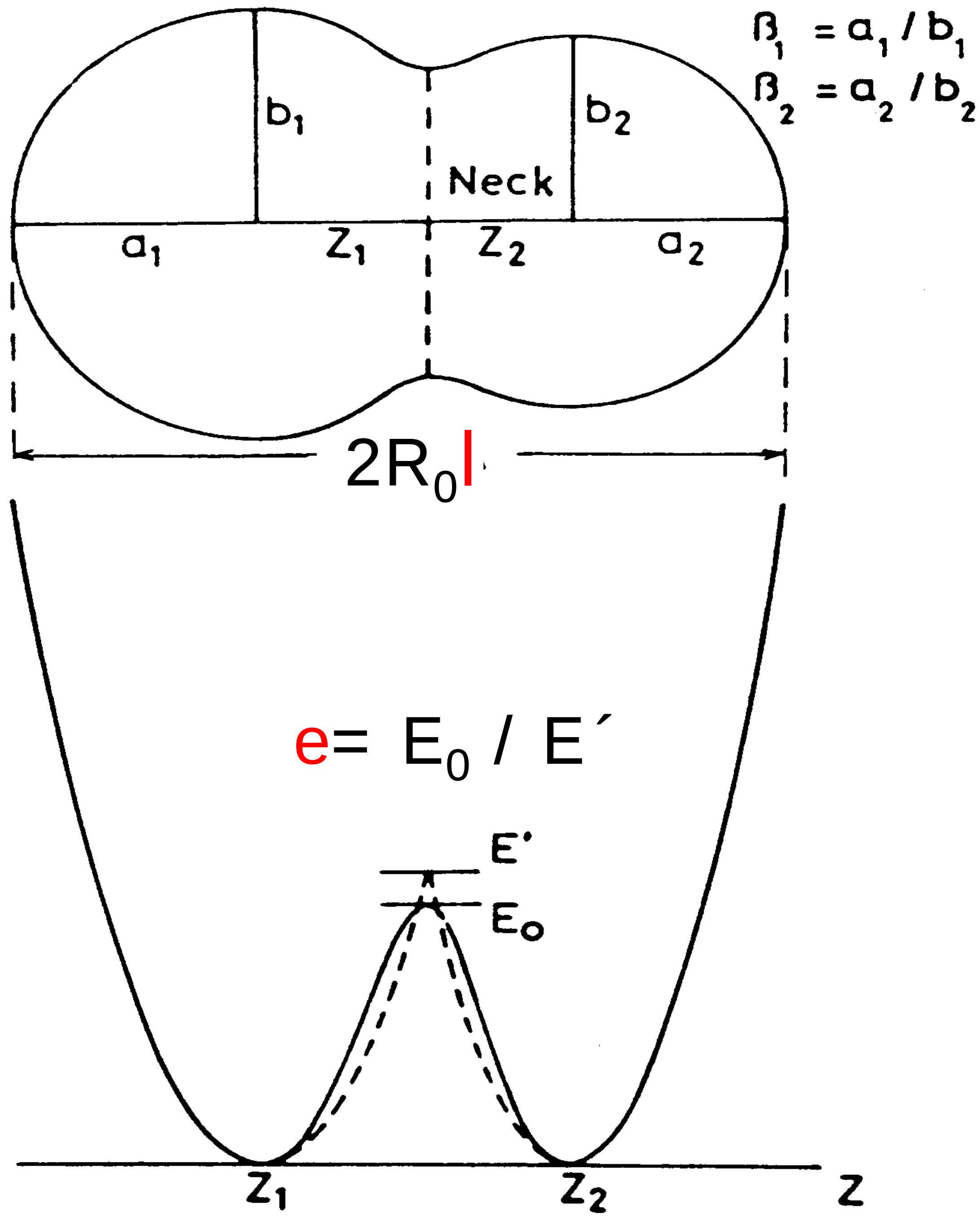
dynamical diabatic in l

4. Study of the neck motion

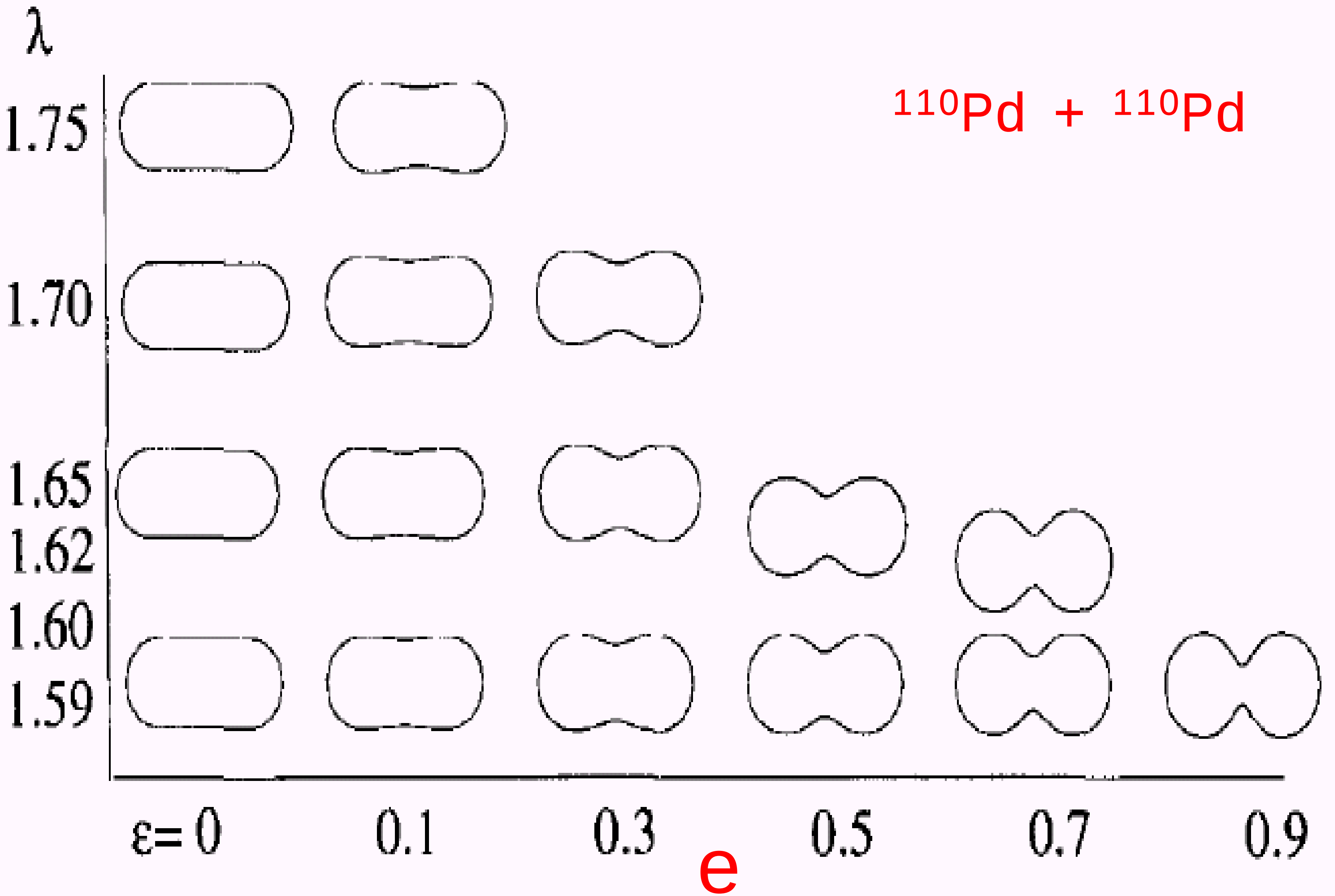
Here, we consider the dynamics of the neck degree of freedom.

The neck parameter $e = E_0 / E'$ is defined by the ratio of the actual barrier height E_0 to the barrier height E' of the two-center oscillator.

The neck grows with decreasing e.



β1 = a1 / b1
β2 = a2 / b2
b1
b2
Neck
a1
Z1
Z2
a2
2R0l
e= E0 / E´
E'
E0
Z1
Z2
Z


λ
1.75
1.70
1.65
1.62
1.60
1.59
ε= 0
0.1
0.3
0.5
0.7
0.9
e
110Pd + 110Pd

We made classical calculations in the coordinates $q_1 = l$ and $q_2 = e$.

Equations of motion are derived from a Lagrangian $L = T - U$

with the kinetic energy $$T = \frac{1}{2}\sum_{ij} B_{ij}\dot{q}_i\dot{q}_j$$

and the potential energy

$U(l, e, h) = U_{\text{liquid drop}}(l, e, h) + dU_{\text{shell}}(l, e, h)$.

We disregard the dependence of dU_{shell} on temperature because only smaller excitation energies of 15-30 MeV are considered.

Also dissipative forces are included with Raleigh dissipation function:

$$\Phi = \frac{1}{2}\sum_{ij} \gamma_{ij}\dot{q}_i\dot{q}_j$$

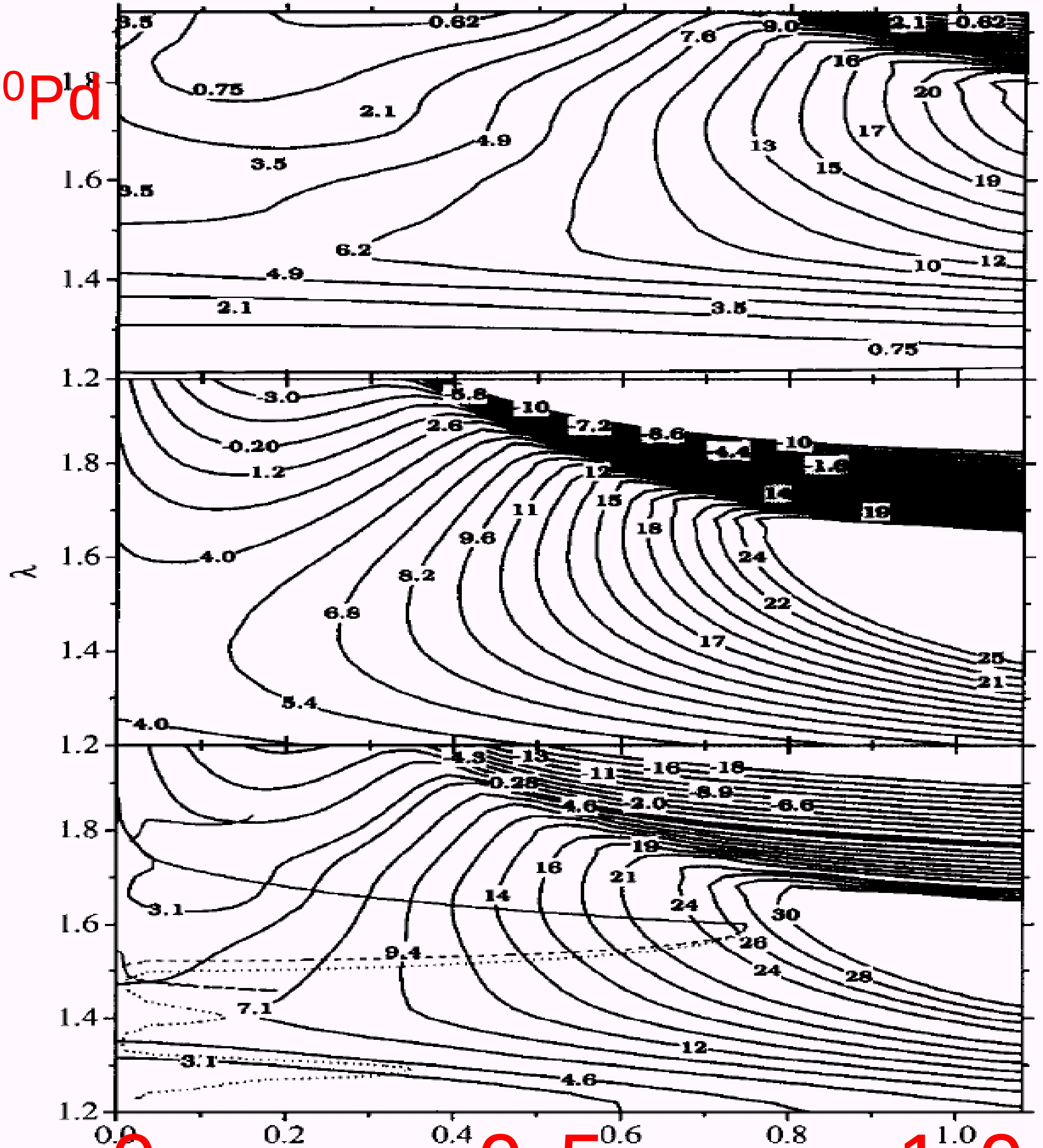
Friction coefficients are caculated with

$$\gamma_{ij} = 2\Gamma B_{ij}/\hbar$$

according to linear response theory; G is the average width of single particle states.

With growing neck the system rapidly falls to the fission-type valley and the fusion occurs due the diffusion of the system in this valley to smaller elongations.

$^{110}Pd + {}^{110}Pd$



$b_i = 1.2$ shell corr.

$b_i = 1.2$ no shell corr.

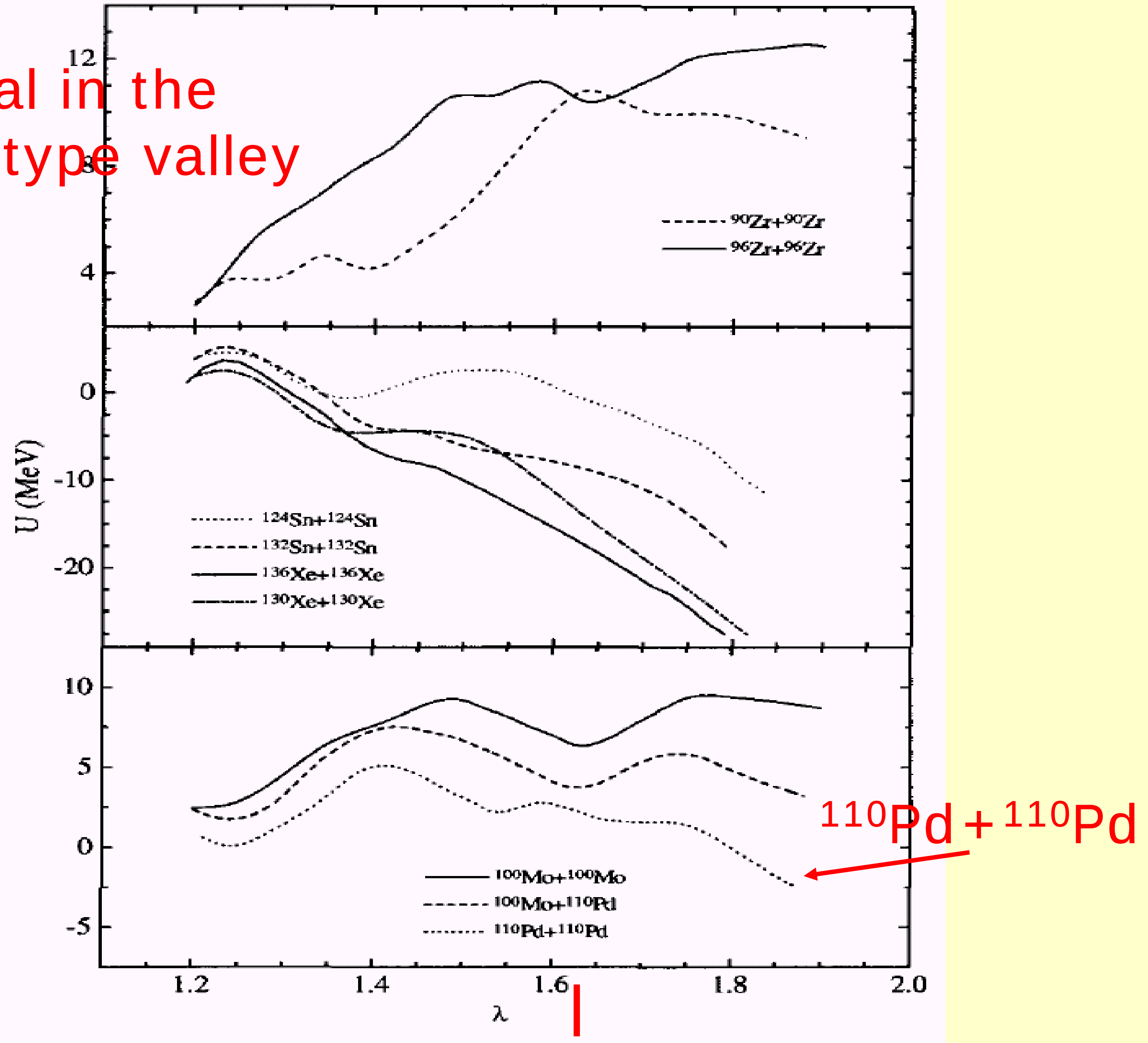
$b_i = 1.0$ shell corr.

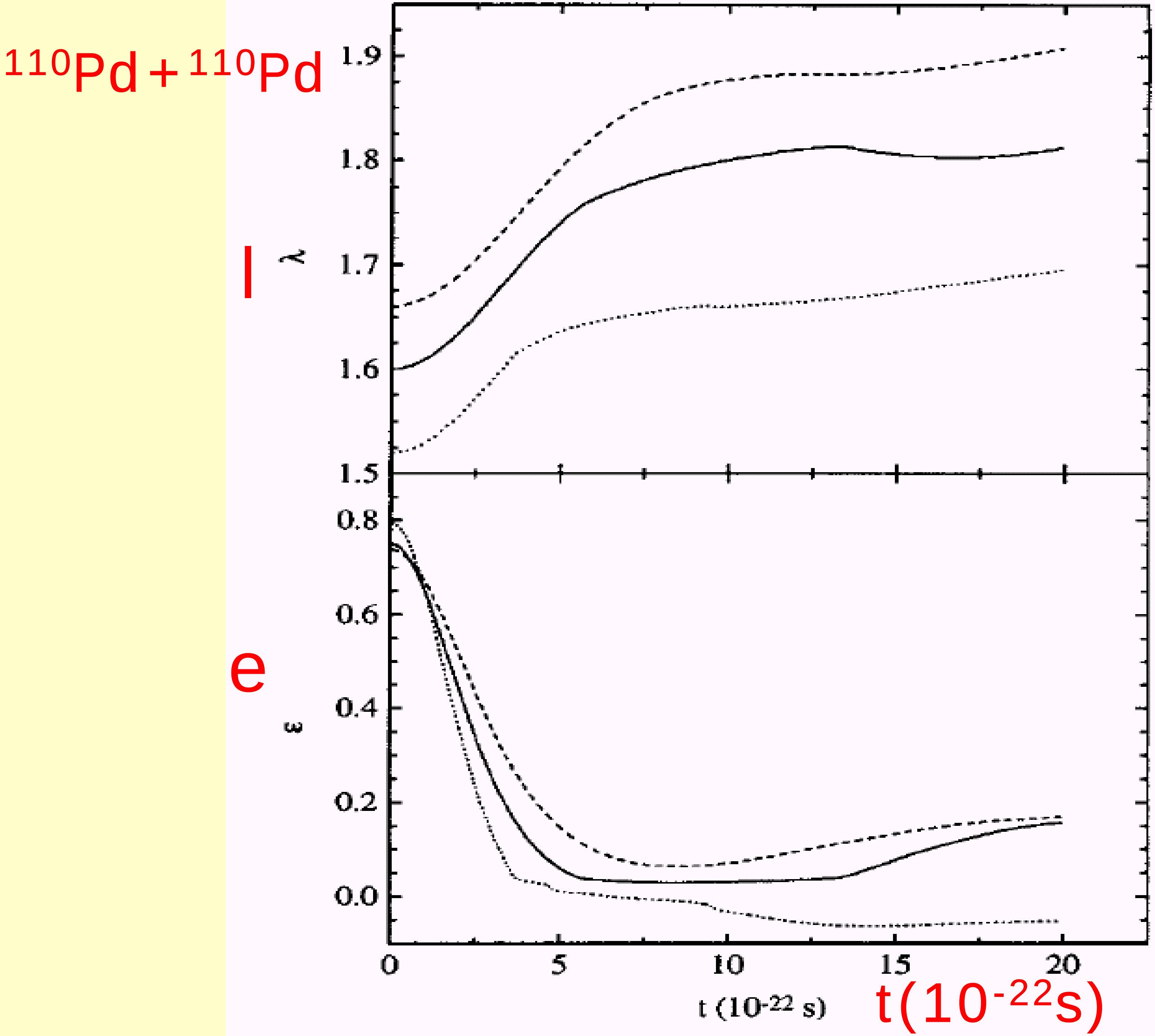
l

e

0 0.5 1.0

Potential in the fission-type valley





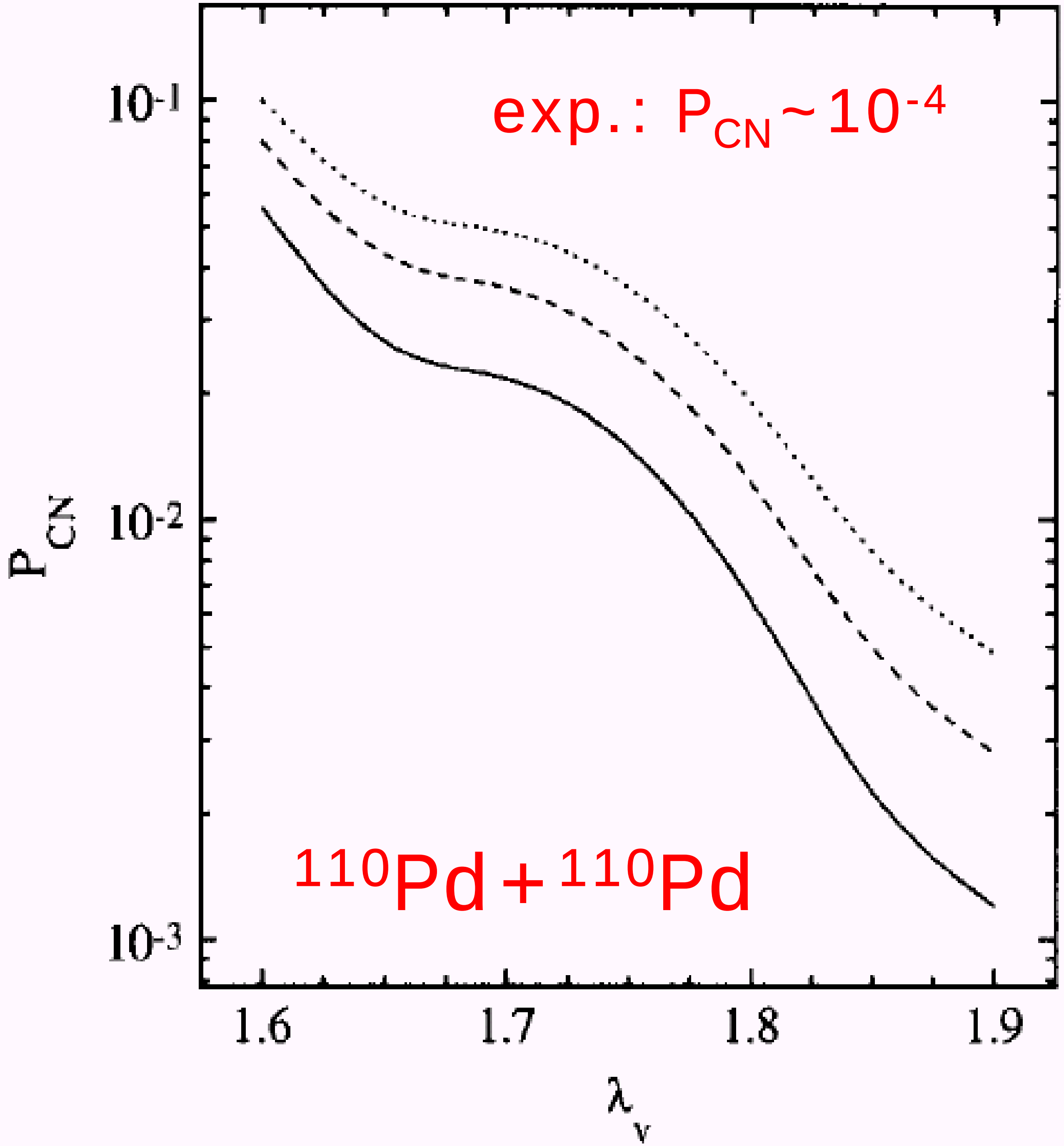
$^{110}Pd + ^{110}Pd$
l
λ
1.9
1.8
1.7
1.6
1.5
e
ε
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
t (10-22 s)
t(10-22s)

Starting with $l = 1.59$, $e = 0.75$ for $^{110}Pd + {}^{110}Pd$

First, mass parameters are obtained with Werner-Wheeler approximation by assuming incompressible and irrotational flow.

Fission-type valley reached in very short time of 3-4 x 10^{-22}s with $l \sim 1.68$, then oscillations in this valley in case of small kinetic energies. Characteristic time of all processes is ~ 5 x 10^{-21}s.

Fusion would occur easier in reactions with heavier isotopes; contradiction to experimental data.



20 MeV
10 MeV

0 MeV

above
Bass
barrier

Wrong dependence of fusion probability on the isotope composition and mass asymmetry.

There must exist a hindrance for a fast growth of the neck and the motion to smaller l .

Essential hindrance: Large microscopically calculated mass parameters for e motion.

Main contributions to $B_{ij}^{Cranking}$ result from

$$B_{ij}^{Cranking} \approx \hbar^2 \sum_{\alpha} \frac{f_\alpha}{\Gamma_\alpha^2} \frac{\partial E_\alpha}{\partial q_i} \frac{\partial E_\alpha}{\partial q_j} \quad \text{with} \quad f_\alpha = -\frac{dn_\alpha}{dE_\alpha}$$

E_a, n_a are TCSM single-particle eigenvalues and occupation numbers, G_a width of decaying single-particle states.

We found much larger neck mass parameter

$B_{ll}^{cr} = B_{ll}^{WW}$, B_{ee}^{cr} » $30\ B_{ee}^{WW}$, B_{le}^{cr} » $0.35\ B_{le}^{WW}$

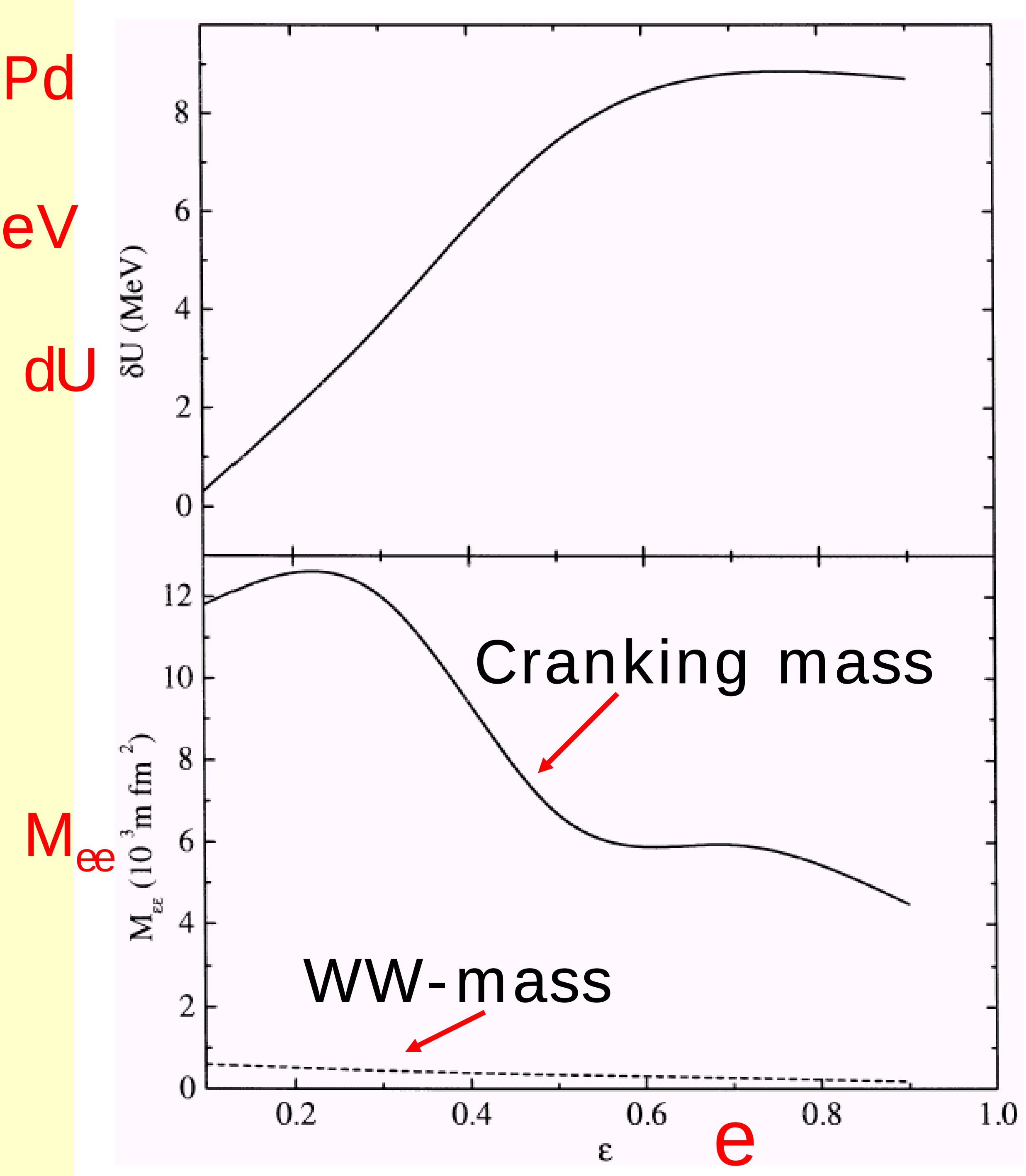
Much larger neck mass parameter than in Werner-Wheeler approximation. System stays near the entrance configuration (DNS - configuration) for a sufficiently long time.

Then thermal fluctuations are responsible for the fusion in the DNS – configuration.

$^{110}Pd + ^{110}Pd$

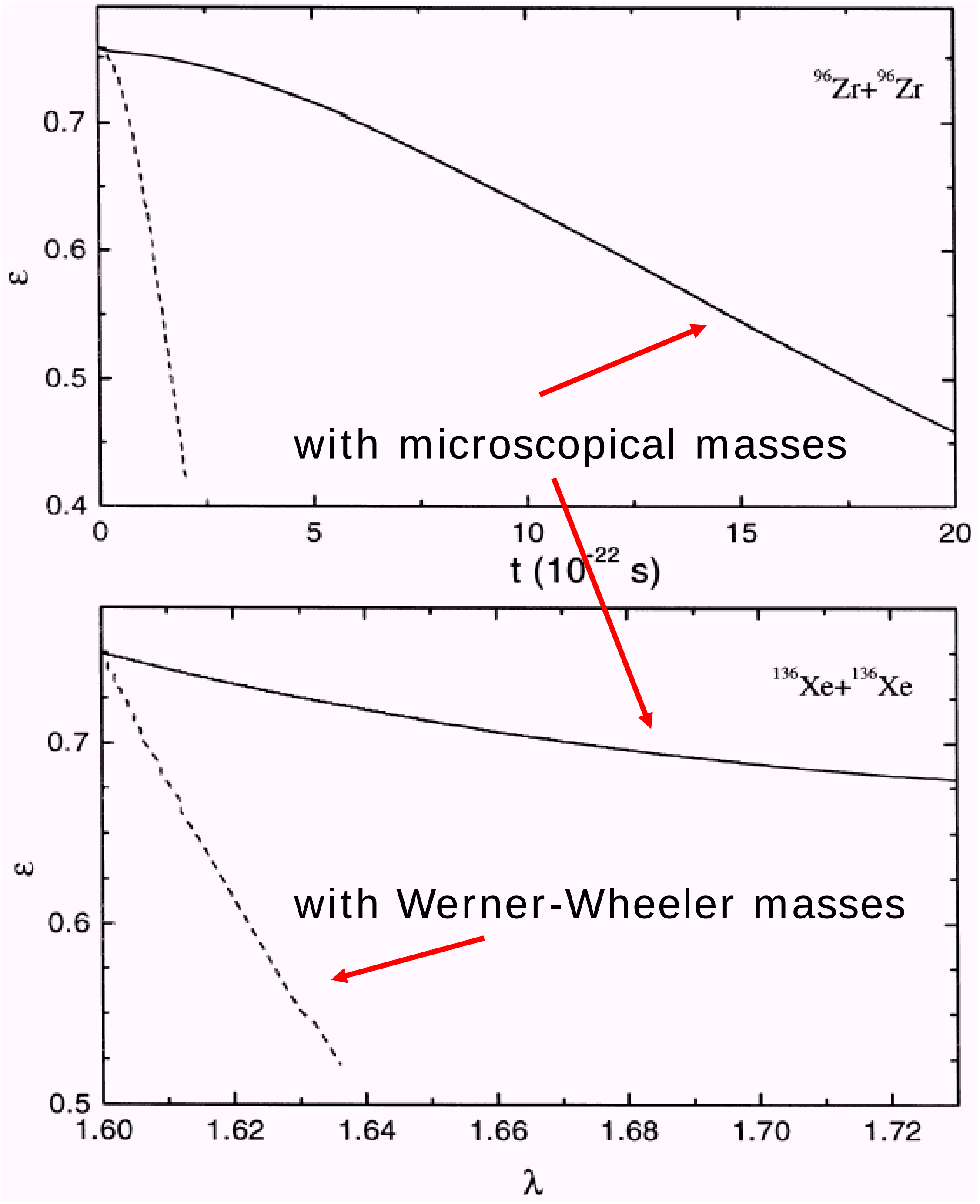
l = 1.6,

E* = 30 MeV



e

e



96Zr+96Zr
0.7
0.6
0.5
0.4
ε
0
5
10
15
20
t (10-22 s)
with microscopical masses
136Xe+136Xe
0.7
0.6
0.5
ε
1.60
1.62
1.64
1.66
1.68
1.70
1.72
λ
with Werner-Wheeler masses

The calculations show a slow growth of the neck. The results justify the assumption of a fixed neck as applied in the DNS model.

5. Repulsive potential by quantization

a) General consideration

(Fink and Greiner 1975)

The energy of a nucleus-nucleus system

$$H = T(x^i, \dot{x}^i) + V(x^i), \qquad x^i = x^1, x^2, x^3 \ldots$$

$$T = \frac{1}{2} g_{ik} \dot{x}^i \dot{x}^k$$

quantization: $$\hat{T} = -\frac{\hbar^2}{2} g^{-1/2} \frac{\partial}{\partial x^i} g^{ik} g^{1/2} \frac{\partial}{\partial x^k}$$

with

$$g = \det(g_{ik}), \quad g^{ik} = (g^{-1})_{ik}, \quad g^{ik} g_{kl} = \delta_{il}$$

Assumption: $x^1=R$,

$x^{m=2,3,4\ldots}$ = other coordinates (Greek letters);

after some transformations:

$$H = -\frac{\hbar^2}{2} g^{11} \frac{\partial^2}{\partial R^2} - \frac{\hbar^2}{2} g^{-1/2} \frac{\partial}{\partial x^\mu} \tilde{g}^{\mu\nu} g^{1/2} \frac{\partial}{\partial x^\nu} + V(R, x^\mu + \int_\infty^R \frac{g^{1\mu}}{g^{11}} dR') + V_{add}(R)$$

with

$$V_{add} = \frac{\hbar^2}{4} g^{11} \left(\frac{\partial^2}{\partial R^2} \ln(g^{11} g^{1/2}) + \frac{1}{2} \left(\frac{\partial}{\partial R} \ln(g^{11} g^{1/2}) \right)^2 \right)$$

Change of potential and an additional potential V_{add}

b) Example: $^{12}C + {}^{12}C$ scattering

Coord.: $\mathbf{r} = \mathbf{R}$ relative motion,

$a^{(1)}_{2m}$, $a^{(2)}_{2m}$ quadrupole deformations of nuclei

Advantage to use symmetrical and antisymmetrical coordinates in this case

$$a^{(s)}_{2m} = \frac{1}{\sqrt{2}}(a^{(1)}_{2m} + a^{(2)}_{2m})$$

$$a^{(a)}_{2m} = \frac{1}{\sqrt{2}}(a^{(1)}_{2m} - a^{(2)}_{2m})$$

$$
\begin{aligned}
H = & - \frac{\hbar^2}{2} g^{11} \frac{\partial^2}{\partial r^2} + \frac{\hbar^2}{2\Theta} \hat{L}^2 \\
& + \frac{\hbar^2}{2i} D\sqrt{5} \left[\left[Y_2 \otimes \hat{L} - \hat{L} \otimes Y_2 \right]^{[2]} \otimes \pi_2^{(s)*} \right]^{[0]} \\
& + \frac{1}{2} \sum_{J=0,2,4} (2J+1)^{1/2} C_J^{(s)} \left[\left[\pi_2^{(s)*} \otimes \pi_2^{(s)*} \right]^{[J]} \otimes Y_J \right]^{[0]} \\
& + \frac{1}{2} \sum_{J=0,2,4} (2J+1)^{1/2} C_J^{(a)} \left[\left[\pi_2^{(a)*} \otimes \pi_2^{(a)*} \right]^{[J]} \otimes Y_J \right]^{[0]} \\
& + V \left(\boldsymbol{r}, \alpha_{2\mu}^{(a)}, \alpha_{2\mu}^{(s)} - \left(\frac{4\pi}{5} \right)^{(1/2)} \beta(r) Y_{2\mu} \right) + V_{add}(r)
\end{aligned}
$$

inverse radial mass $g^{11} = 1/\mu(r)$

moment of inertia $Q(r)$

angular momentum operator of relative motion L

Transformation of HΨ = EΨ to a constant mass m_0 by multiplying the Schrödinger equation with $1/\left(g^{11}\cdot\mu_0\right)$:

$$H' = -\frac{\hbar^2}{2\mu_0}\frac{\partial^2}{\partial r^2} + \ldots \frac{1}{g^{11}\mu_0}V_{add} + \frac{g^{11}\mu_0 - 1}{g^{11}\mu_0}E$$

$\downarrow$ V_{add} $\qquad$ $\downarrow$ $V(E)$

V_{add} is essentially generated by the coupling of the a_{2m} - degrees of freedom to the relative motion.

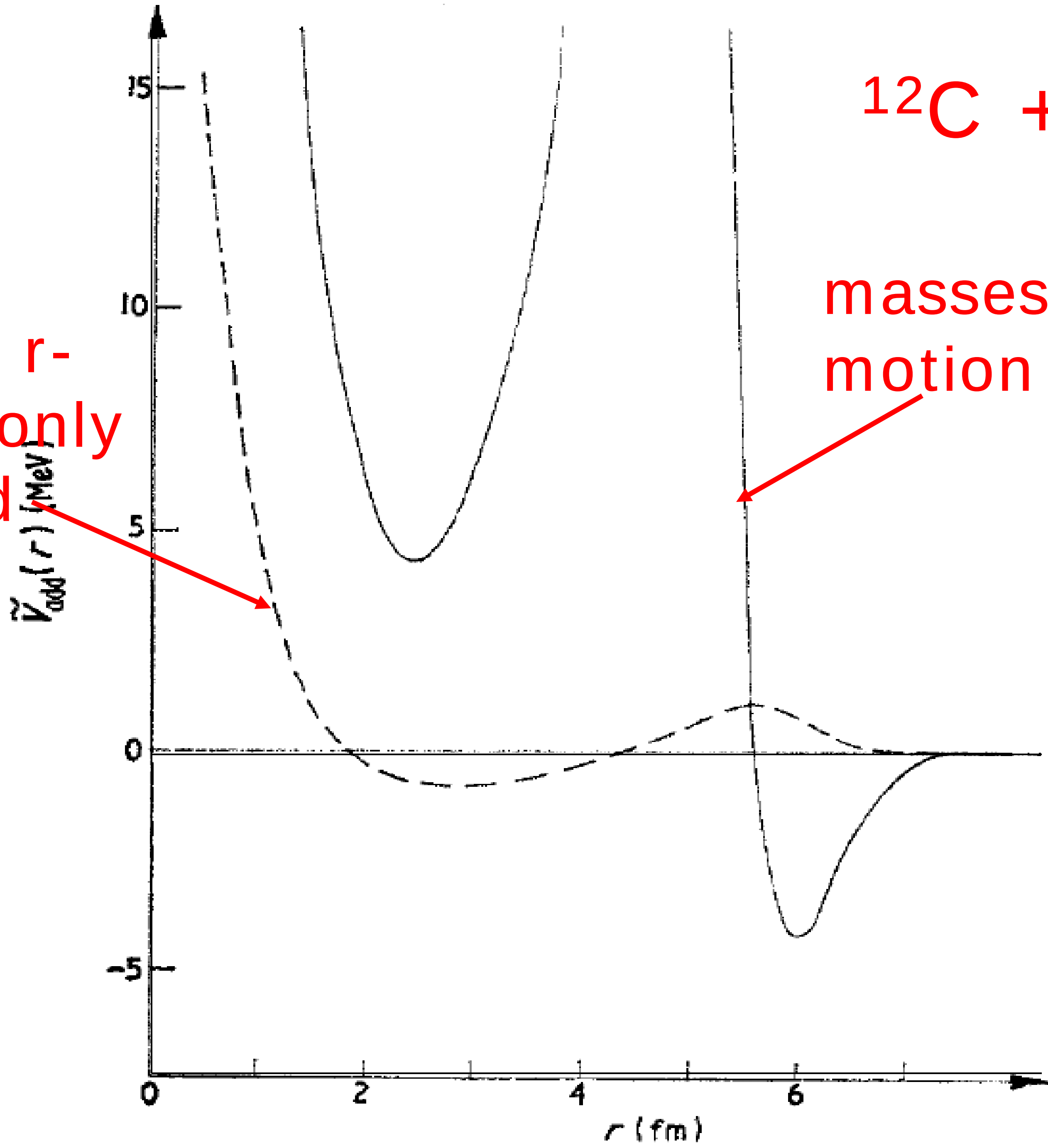
Correct inclusion of more degrees of freedom yields repulsive potentials.

V_{add}

$^{12}C + ^{12}C$

mass of r-motion only included

masses of a$_{2\ m}$-motion included



r (fm)

6. Summary and conclusions

Fusion reations for the production of superheavy nuclei are explained with adiabatic and diabatic potentials.

The dynamics of fusion is very different in the case of adiabatic and diabatic potentials:

In adiabatic potentials the nuclei melt together along the internuclear distance. This yields larger fusion cross sections for symmetric target and projectile combinations in contradiction to known experimental data.

Since diabatic potentials are repulsive, the nuclei form a dinuclear system of two touching nuclei and exchange nucleons up to the point when the compound nucleus is formed. This yields smaller fusion cross sections for symmetric target and projectile combinations in agreement with the experimental data.

The formation of a larger neck is hindered by a large, microscopically calculated mass parameter for the neck degree of freedom.

What is the “correct“ answer for the question:

Melting or Transfer of nucleons in the production of superheavy nuclei?

Comparing the experimental data and many calculations I must conclude that the dinuclear model gives correct predictions. The dinuclear model is based on the transfer of nucleons and can explain the production of superheavy nuclei with it.

D.G.